M01T.3

a). We Choose the gauge that $\vec{A} = (0, Bx, 0)$. The Hamiltonian is

$$H = \frac{p_x^2}{2m} + \frac{(p_y + eBx)^2}{2m} + \frac{p_z^2}{2m} = \frac{p_x^2}{2m} + \frac{1}{2} m\omega_L^2 (x - x_0) + \frac{p_z^2}{2m} \tag{1}$$

, where $\omega_L = \frac{eB}{m}$, $x_0 = \frac{-p_y}{m\omega_L}$. k_z and k_y are good quantum numbers.

In z and y direction, the motion is described by plane waves, with $E_z = \frac{p_z^2}{2m}$. While in x direction, motion is described by harmonic oscillator, whose center is determined by p_y. The in-plane degree energy is

$$E_n = (n + 1/2)\hbar\omega_L \tag{2}$$

, the so called Landau Levels, where n is integer. The center of x-direction motion should be within the plane, thus, the degeneracy of E_n is calculated as

$$g = \frac{L_x}{2\pi\hbar/L_y/(m\omega_L)} = \frac{\Phi}{\Phi_0} \tag{3}$$

, due to quantization of $k_y = n_y \frac{2\pi}{L_y}$, the result of periodic condition. Here $\Phi = BL_xL_y$, the total flus, $\Phi_0 = h/e$ the flux quanta.

b). In dilute case, the overlap of the wave function of particles are negligible, so they can be viewed as identical classical particles. The grand partition function is $Z = Exp[e^{\beta\mu} z]$, where the small z is the canonical partition function for single particle.

$$\ln Z = e^{\beta\mu} \sum_{k_z, n} g e^{-(n+1/2)\beta\hbar\omega_L - \beta E_{kz}} = \frac{BV}{\Phi_0 \lambda} \frac{e^{\beta\mu}}{2\sinh(\frac{1}{2}\hbar\omega_L\beta)} \tag{4}$$

$$\lambda = \frac{h}{\sqrt{2\pi m k_B T}} \tag{5}$$

In the derivation above, I have used

$$\sum_{k_z} e^{-\beta\hbar^2 k_z^2/2m} = \sum_{n_z} e^{-\beta\hbar^2 (\frac{2\pi}{L_z})^2 n_z^2/2m} = \frac{L_z}{2\pi} \int_0^\infty dk_z e^{-\beta\hbar^2 k_z^2/2m} = \frac{L_z}{\lambda} \tag{6}$$

$$\sum_n e^{-\beta(n+1/2)\hbar\omega_L} = \frac{1}{2\sinh(\frac{1}{2}\hbar\beta\omega_L)} \tag{7}$$

The pressure is

$$P = \frac{1}{\beta} \frac{\partial \ln Z}{\partial V} = \frac{B}{\beta\Phi_0 \lambda} \frac{e^{\beta\mu}}{2\sinh(\frac{1}{2}\hbar\omega_L\beta)} \tag{8}$$

3). We can get

$$N = \frac{1}{\beta} \frac{\partial \ln Z}{\partial \mu} = \ln Z \tag{9}$$

$$M = \frac{1}{\beta} \frac{\partial \ln Z}{\partial B} \tag{10}$$

$$= \frac{\rho V}{\beta} [\frac{1}{B} - \frac{\hbar e \beta \cosh(\frac{1}{2}\hbar\omega_L\beta)}{2m \sinh(\frac{1}{2}\hbar\omega_L\beta)}] \tag{11}$$

In deriving the magnetism and susceptibility, we should taylor expand the term $\sinh(\beta\omega_L\hbar/2)$as to B to third order, and keep in mind that B is a small quantity. Term like $\frac{B}{2\sinh CB}$ (C= constant) is expanded as $\frac{B}{2\sinh CB} -> \frac{B}{CB+\frac{1}{3}(CB)^3} = \frac{1}{C+C^3B^2/3} = \frac{1}{C}\left(1-\frac{1}{3}C^2B^2\right)$. Then, second derivative gives you $\frac{\partial^2}{\partial B^2} \frac{B}{2\sinh CB} -> \frac{-2C}{3}$.

Susceptibility is $\chi = \frac{1}{\beta} \frac{\partial^2 \ln Z}{\partial B^2} = -\frac{Ve^2 e^{\beta\mu}}{24\pi\lambda m} = -\frac{N\beta}{12} (\frac{\hbar^2 e^2}{m^2})^2 < 0$.

So, it is diamagnetic.

2 thoughts on “M01T.3”

M
December 17, 2013 at 1:48 am

OK, it looks like a good solution. Especially if you can do this in no more than 45 minutes.

Now it makes sense to expand certain parts and fix minor mistakes.
Fix a typo (autocorrection?) in the very first line of the solution. Add missing $\hbar$ in (2). Write an explicit answer for the energy levels in part (a) -- you have all ingredients already.
In (b): I'd say that (3) needs more explanation -- how do you sum over k_z?
Also I suggest expanding part (c) a little bit by adding some skipped steps.

J
December 17, 2013 at 12:15 pm

Hi, M, I have added some details and correct the errors.