# 1 May 2001, Thermodynamics, Problem 1

## 1.1 (a)

Photons have energy given by:

$$E = h\nu$$

Then you can compute the internal energy of a photon gas, which is something like:

$$U = \int g(\nu)n(\nu)d\nu = \frac{V(k_BT)^4}{c(\hbar c)^3}I$$

where $n(\nu)$ is the Bose-Einstein distribution. Notice that the maximum in the energy spectrum will be for the frequency that maximizes the integrand. You can differentiate it, set it equal to 0 and solve it numerically to obtain:

$$h\nu_{max} = c_1kT$$

Use this to solve for h and plug it into the equation for U. You will get:

$$U = \int g(\nu)n(\nu)d\nu = \frac{V(k_BT)^4}{c(2\pi\frac{c_1T}{\nu_{max}}c)^3}I$$

Now if you have a photon gas at temperature T, you can measure the energy and find the frequency that maximizes power output, that will be $\nu_{max}$. Measure the volume, compute the integral I and measure the average energy. You can then solve for $k_B$. Finally obtain Avogadro's number from $A = R/k_B$. And to get h, solve the maximum frequency condition for h and you are done.

## 1.2 (b)

$$C_V(T) = \left(\frac{\partial U}{\partial T}\right)_V \approx \frac{V(k_BT)^3k_B}{c(\hbar c)^3}I$$

Also, when you measure a heat input, you may get an extra constant in the heat capacity, but by Nernst's third law you have to get a heat capacity that goes to 0 faster than T as T goes to 0. You measure $C_V$, V, T, $k_B$ and c, and obtain $h = 2\pi\hbar$.