1 May 2001, Thermodynamics, Problem 1

1.1 (a)

Photons have energy given by:

\[ E = h \nu \]

Then you can compute the internal energy of a photon gas, which is something like:

\[ U = \int g(\nu)n(\nu)d\nu = \frac{V(k_B T)^4}{c(hc)^3} I \]

where \( n(\nu) \) is the Bose-Einstein distribution. Notice that the maximum in the energy spectrum will be for the frequency that maximizes the integrand. You can differentiate it, set it equal to 0 and solve it numerically to obtain:

\[ h\nu_{\text{max}} = c_1 kT \]

Use this to solve for \( h \) and plug it into the equation for \( U \). You will get:

\[ U = \int g(\nu)n(\nu)d\nu = \frac{V(k_B T)^4}{c(2\pi \frac{c_1}{\nu_{\text{max}} c})^3} I \]

Now if you have a photon gas at temperature \( T \), you can measure the energy and find the frequency that maximizes power output, that will be \( \nu_{\text{max}} \). Measure the volume, compute the integral \( I \) and measure the average energy. You can then solve for \( k_B \). Finally obtain Avogadro’s number from \( A = R/k_B \). And to get \( h \), solve the maximum frequency condition for \( h \) and you are done.

1.2 (b)

\[ C_V(T) = \left( \frac{\partial U}{\partial T} \right)_V \approx \frac{V(k_B T)^3 k_B}{c(hc)^3} I \]

Also, when you measure a heat input, you may get an extra constant in the heat capacity, but by Nernst’s third law you have to get a heat capacity that goes to 0 faster than \( T \) as \( T \) goes to 0. You measure \( C_V, V, T, k_B \) and \( c \), and obtain \( h = 2\pi \hbar \).