1 May 2001, Mechanics, Problem 3

1.1 (a)

$$\boldsymbol{F} = mg\hat{j} + q\dot{\boldsymbol{x}} \times B\hat{k} = m\ddot{\boldsymbol{x}}$$

The Russians' solution to this problem is clever and easy to understand, so I won't go into detail with my solution, just write the solutions and clarify points:

$$x(t) = \frac{mgt}{qB} - \frac{m^2 gsin(qBt/m)}{(qB)^2} \tag{1}$$

$$y(t) = \frac{m^2 g[1 - cos(qBt/m)]}{(qB)^2} \tag{2}$$

The particle moves along the positive x direction while circling around. So it looks like a helix.

1.2 (b)

The equation is modified to:

$$\boldsymbol{F} = mg\hat{j} + q\dot{\boldsymbol{x}} \times B\hat{k} - \beta\dot{\boldsymbol{x}} = m\ddot{\boldsymbol{x}}$$

The (not so nice) solution is:

$$x(t) = \frac{mgtqB}{\beta^2 + (qB)^2} + \frac{m^2 g\{-e^{-\beta t/m} sin(\frac{qBt}{m})[(qB)^2 - \beta^2] - [1 - e^{-\beta t/m} cos(\frac{qBt}{m})]2\beta qB\}}{[\beta^2 + (qB)^2]^2} \tag{3}$$

$$y(t) = \frac{\beta mgt}{\beta^2 + (qB)^2} + \frac{m^2 g\{[1 - e^{-\beta t/m} cos(\frac{qBt}{m})][(qB)^2 - \beta^2] - e^{-\beta t/m} sin(\frac{qBt}{m})2\beta qB\}}{[\beta^2 + (qB)^2]^2} \tag{4}$$

Taking the time derivative of these equations and letting t go to infinity we see:

$$\boldsymbol{v}_{final} = \frac{mg}{\beta^2 + (qB)^2}(qB\hat{x} + \beta\hat{y}) \tag{5}$$

1.3 (c)

The motion from part a was a superposition of a shifting and circling. When we include the effect of electromagnetic radiation, it is the circling that counts, because electromagnetic radiation arises from acceleration of charged particles. Therefore, that velocity will be diminishing until in the end there is no more circling, and the particle just moves in the x direction with velocity mg/qB.