M01E.3

M01E.3 - Charged Rotating Cylindrical Shells

1. As the charge density is constant on the cylinder

surfaces. Electric filed must obey the symmetric law and can only be in radial direction and has no dependence on ϕ. Choose a cylinderical Gauss-box

$$2\pi r L \epsilon_0 E = \begin{cases} 0 & r \leq a, \\ 2\pi a L \sigma_a & a < r < b, \\ 2\pi L(a\sigma_a - b\sigma_b) & r \geq b. \end{cases} \tag{1}$$

Thus

$$E(r) = \begin{cases} 0 & r \leq a, \\ a\sigma_a/\epsilon_0 r & a < r < b, \\ (a\sigma_a - b\sigma_b)/\epsilon_0 r & r \geq b. \end{cases} \tag{2}$$

2.To ensure $\vec{E} = 0$ outside the outer cylinder,

$$a\sigma_a = b\sigma_b \tag{3}$$

3. As this cylinder is very long and thin, we can apply the model of solenoid to it,

$$\begin{aligned} B &= \mu_0 n I \\ &= \mu_0 \frac{2\pi\sigma_b b}{2\pi/\omega_b} \\ &= \mu_0 \omega_b \sigma_b b \end{aligned} \tag{4}$$

4. To ensure $\vec{B} = 0$ inside of inner cylinder,

$$\mu_0 \omega_b \sigma_b b = \mu_0 \omega_a \sigma_a a \tag{5}$$

Combined with equation (3),

$$\omega_a = \omega_b \tag{6}$$

5. According to Faraday's Law,

$$\oint \vec{E} d\vec{l} = -\frac{\partial}{\partial t} \int \vec{B} d\vec{s} \tag{7}$$

$$2\pi r E = \pi(b^2 - a^2)\mu_0 \sigma_b b \frac{d\omega}{dt} \tag{8}$$

$$E = \frac{(b^2 - a^2)}{2r} \mu_0 \sigma_b b \frac{d\omega}{dt} \tag{9}$$

Thus, the additional external torque is,

$$\vec{N} = \int \vec{r} \times \sigma_b \vec{E} ds \tag{10}$$

$$= \mu_0 \pi L (b^2 - a^2)(b\sigma_b)^2 \frac{d\omega}{dt} \tag{11}$$

6. Angular momentum in the electromagnetic field,

$$\vec{L}_{EM} = \epsilon_0 \int \vec{x} \times (\vec{E} \times \vec{B}) d^3 x \tag{12}$$

$$= \epsilon_0 \int_a^b r \frac{\sigma_b b}{\epsilon_0 r} \mu_0 \omega_b b \sigma_b 2\pi r L dr \hat{\vec{z}} \tag{13}$$

$$= \pi L \mu_0 \omega_b (\sigma_b b)^2 (b^2 - a^2) \hat{\vec{z}} \tag{14}$$

Integral the additional external torque calculated in part 5 over time,

$$\Delta\vec{L} = \int \vec{N} dt = \pi L \mu_0 \omega_b (\sigma_b b)^2 (b^2 - a^2)\hat{\vec{z}} \tag{15}$$

$$= \vec{L}_{EM} \tag{16}$$

One thought on “M01E.3”

December 15, 2013 at 8:10 pm

I think in (8) and in (9) there should be r^2 instead of b^2 on the right-hand side.

Otherwise, everything is OK.
