Two identical plates of length $c$ and width $d$ are separated by an angular separation of $\phi_0$, and the nearest point on each plate is a distance $b$ from the origin. The plate at $\phi = 0$ is grounded, and the plate at $\phi = \phi_0$ is set at potential $V$.

a. Compute the stored energy in the capacitor. Assume that the electrical potential between the plates depends only on $\phi$, and ignore fringe fields. (In which limit is this an allowed approximation?).

Now take ten plates in a cylindrical arrangement, and connect the odd plates together with one wire, and the even plates with another. There is no direct connection between the odd and even plates. Assume a charge $Q$ is placed on the even plates, and a charge $-Q$ on the odd plates.

b. Compute the total capacitance of this structure.

Since the potential $\Phi$ depends only on the angle $\phi$, the general solution for the potential will be:

$$\Phi = A + B\phi$$

To fit the boundary conditions, we then have:

$$\Phi = \frac{V}{\phi_0}\phi$$

We can then find the electric field:

$$E = -\nabla \cdot \Phi$$
\[ E_\rho = 0; \quad E_\phi = -\frac{1}{\rho} \frac{V}{\phi_0}; \quad E_z = 0 \]

The surface charge density is given by the electric field:

\[ \sigma = \varepsilon_0 E_\phi = -\frac{1}{\rho} \frac{\varepsilon_0 V}{\phi_0} \]

So the total charge on one plate is given by:

\[ Q = -d \int_{b}^{b+c} \frac{1}{\rho} \frac{\varepsilon_0 V}{\phi_0} d\rho = -\frac{d\varepsilon_0 V}{\phi_0} \ln \left( \frac{b+c}{b} \right) \]

and so the stored energy is:

\[ W = \frac{1}{2} QV = \frac{d\varepsilon_0 V^2}{2\phi_0} \ln \left( \frac{b+c}{b} \right) \]

The approximation will be valid for small \( \phi \) (so that there is no dependence on \( \rho \)) and for \( c, d \gg b \) (to neglect edge effects).

From part a:

\[ Q = -\frac{d\varepsilon_0 V}{\phi_0} \ln \left( \frac{b+c}{b} \right) \]

So that:

\[ V = \frac{Q\phi_0}{d\varepsilon_0} \frac{1}{\ln \left( \frac{b+c}{b} \right)} \]

The capacitance is then, using \( \phi_0 = \pi / 5 \):

\[ C = \frac{Q}{V} = \frac{5d\varepsilon_0}{\pi} \ln \left( \frac{b+c}{b} \right) \]

Since we have ten plates, we then get:

\[ C = \frac{Q}{V} = \frac{50d\varepsilon_0}{\pi} \ln \left( \frac{b+c}{b} \right) \]

(One can think of this as each even plate having charge 2Q at voltage V, since it is charged on both sides, and then counting 5 of these plates and plugging into part a).