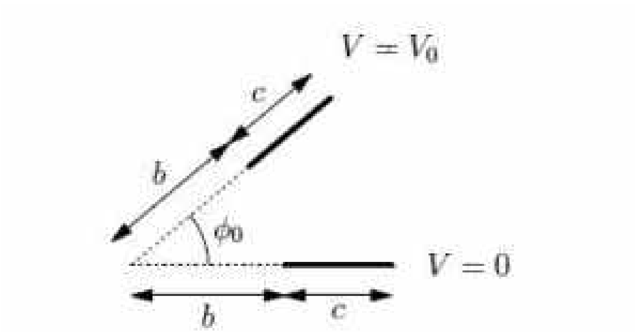
M01E.1

Solution to M01E.1 — Non-Parallel Plate Capacitor



The approximation for which the fringing fields can be ignored is valid when the radial length of the plates, c, is very large.

The only relevant term of the Laplacian (in cylindrical coordinates) will be be

$$\frac{1}{s^2}\frac{\partial^2 V}{\partial \phi^2} \tag{1}$$

by the stated assumption that $V(\phi)$. From this follows,

$$\frac{1}{s^2}\frac{\partial^2 V}{\partial \phi^2} = 0 \tag{2}$$

$$\frac{\partial^2 V}{\partial \phi^2} = 0 \tag{3}$$

$$V(\phi) = C_1 \phi + C_2 \tag{4}$$

Utilizing the boundary conditions of $V(0) = 0$ and $V(\phi_0) = V_0$, we find that

$$V(\phi) = \frac{V_0 \phi}{\phi_0} \tag{5}$$

Solving for the charge density on the plates

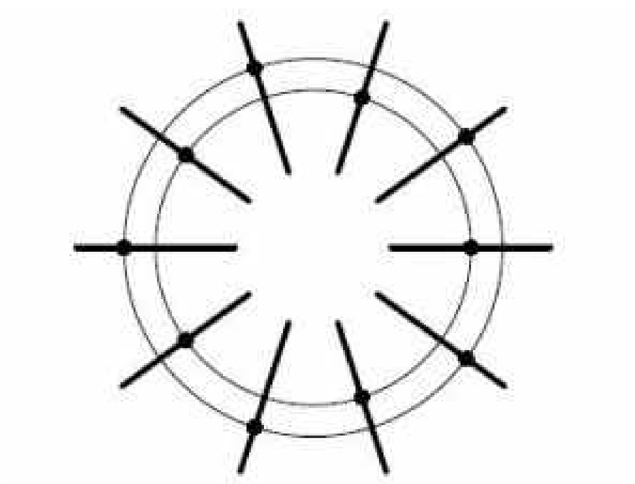
$$\hat{\phi} \cdot (\nabla V) = \frac{1}{s}\frac{\partial V}{\partial \phi} = \frac{\sigma}{\epsilon_0} \Rightarrow \sigma = \frac{\epsilon_0 V_0}{s\phi_0} \tag{6}$$

Integrating over the plates (including the provided height, z) to find the total charge Q

$$Q = \int \sigma\, dA \Rightarrow Q = \int_0^d dz \int_b^b +c \frac{\epsilon_0 V_0}{s\phi_0}\, ds = \frac{\epsilon_0 V_0 d}{\phi_0} \ln\left(\frac{b+c}{b}\right) \tag{7}$$

So expressing the energy in terms of the total charge,Q, and the potential difference between the plates,V_0 , we see that

$$U = \frac{1}{2}\left[\frac{\epsilon_0 V_0^2 d}{\phi_0} \ln\left(\frac{b+c}{b}\right)\right] \tag{8}$$



For the given arrangement of plates, this will be the equivalent of 10 of our original capacitors placed in series so the total capacitance will be the sum of the individual capacitances, which by symmetry have the same value. Using

$$C = \frac{Q}{V} = \frac{\frac{\epsilon_0 V_0 d}{\phi_0} \ln\left(\frac{b+c}{b}\right)}{\frac{V_0 \phi}{\phi_0}} = \frac{\epsilon_0 d}{\phi} \ln\left(\frac{b+c}{b}\right) \tag{9}$$

Because the 10 plates are evenly arranged around a circle,

$$\phi = \frac{2\pi}{10} = \frac{\pi}{5} \tag{10}$$

and so the total capacitance will be

$$C_{eq} = \frac{50\epsilon_0 d}{\pi} \ln\left(\frac{b+c}{b}\right) \tag{11}$$

One thought on “M01E.1”

December 15, 2013 at 7:49 pm

>The approximation for which the fringing fields can be ignored is valid when
>the radial length of the plates, c, is very large.

Not only the radial length, but also the width d. And not just large, but large compared to the plates separation.

Regarding the rest of your solution, all the answers are correct. Note that in part (b) it is NOT a serial connection, it is a parallel connection. But your answer is correct. (note that in (9) there should be no ϕ, every ϕ there is actually ϕ_0)