Seth Dorfman Prelims December 22, 2005

May 2000 QM

3) $|2,2\rangle = |1,1\rangle$

$\uparrow j \quad \uparrow m_j \qquad \uparrow m_1 \quad \uparrow m_2$

$J_-|2,2\rangle = (L^1_- + L^2_-)|1,1\rangle$

$\sqrt{2\cdot3-2\cdot1}\,|2,1\rangle = \sqrt{1\cdot2-1\cdot0}\,|0,1\rangle + \sqrt{1\cdot2-1\cdot0}\,|1,0\rangle$

$2|2,1\rangle = \sqrt{2}\,|0,1\rangle + \sqrt{2}\,|1,0\rangle$

$|2,1\rangle = \frac{1}{\sqrt{2}}(|0,1\rangle + |1,0\rangle)$

$J_-|2,1\rangle = (L^1_- + L^2_-)\frac{1}{\sqrt{2}}(|0,1\rangle + |1,0\rangle)$

$\sqrt{2\cdot3-1\cdot0}\,|2,0\rangle = \frac{1}{\sqrt{2}}\Big[\sqrt{1\cdot2-0\cdot-1}\,|-1,1\rangle + \sqrt{1\cdot2-1\cdot0}\,|0,0\rangle + \sqrt{1\cdot2-1\cdot0}\,|0,0\rangle + \sqrt{1\cdot2-0\cdot-1}\,|1,-1\rangle\Big]$

$\sqrt{6}\,|2,0\rangle = |-1,1\rangle + 2|0,0\rangle + |1,-1\rangle$

$|2,0\rangle = \frac{1}{\sqrt{6}}(|-1,1\rangle + 2|0,0\rangle + |1,-1\rangle$

$\therefore$ $|2,2\rangle = |1,1\rangle$

$|2,1\rangle = \frac{1}{\sqrt{2}}(|0,1\rangle + |1,0\rangle)$

$|2,0\rangle = \frac{1}{\sqrt{6}}(|-1,1\rangle + 2|0,0\rangle + |1,-1\rangle)$

$|2,-1\rangle = \frac{1}{\sqrt{2}}(|0,-1\rangle + |-1,0\rangle)$

$|2,-2\rangle = |-1,-1\rangle$

Since the X particle is origionally unpolarized, it has an equal probability of being in any of these five states

$\therefore$ $\vert m_1\rangle$ state	Wavefunction	$\theta_1 = \frac{\pi}{2}$
$\vert 1\rangle$	$Y_{11} = -\sqrt{\frac{3}{8\pi}}\sin\theta_1 e^{i\phi_1}$	$Y_{11} = -\sqrt{\frac{3}{8\pi}}e^{i\phi_1}$
$\vert 0\rangle$	$Y_{10} = \sqrt{\frac{3}{4\pi}}\cos\theta_1$	$Y_{10} = 0$
$\vert -1\rangle$	$Y_{1-1} = \sqrt{\frac{3}{8\pi}}\sin\theta_1 e^{-i\phi_1}$	$Y_{1-1} = \sqrt{\frac{3}{8\pi}}e^{-i\phi_1}$

$\therefore$ When $\theta_1, \theta_2 = \frac{\pi}{2}$:

$|2,2\rangle = \frac{3}{8\pi}e^{i(\phi_1+\phi_2)}$

$|2,1\rangle = 0$

$|2,0\rangle = \frac{-1}{\sqrt{6}}\frac{3}{8\pi}\left[e^{i(\phi_2-\phi_1)} + e^{i(\phi_1-\phi_2)}\right] = \frac{-2}{\sqrt{6}}\frac{3}{8\pi}\cos(\phi_1-\phi_2)$

$|2,-1\rangle = 0$

$|2,-2\rangle = \frac{3}{8\pi}e^{-i(\phi_1+\phi_2)}$

Thus the angular ~~probability~~ distribution in each state is:

State	Distribution ($\langle\psi\vert\psi\rangle$)
$\vert 2,2\rangle$	$\left(\frac{3}{8\pi}\right)^2$
$\vert 2,1\rangle$	0
$\vert 2,0\rangle$	$\frac{2}{3}\left(\frac{3}{8\pi}\right)^2 \cos^2(\phi_1 - \phi_2)$
$\vert 2,-1\rangle$	0
$\vert 2,-2\rangle$	$\left(\frac{3}{8\pi}\right)^2$

Since there is an equal probability of the X particle being in any of the five states, the total angular probability distribution is proportional to the sum of the five distributions:

$$P(\phi_1, \phi_2) \propto \left(\frac{3}{8\pi}\right)^2 \left(2 + \frac{2}{3}\cos^2(\phi_1 - \phi_2)\right)$$

Note that the probability depends only on $\ell = \phi_1 - \phi_2$, the angle between the directions at which the α-particles are emitted. This result is independent of the choice of quantization axis $\theta_1, \theta_2 = \frac{\pi}{2}$.

$$\therefore P(\ell) = A\left(1 + \frac{1}{3}\cos^2 \ell\right)$$

Normalizing:
$$\frac{1}{A} = 2\pi\left(1 + \frac{1}{3}\cdot\frac{1}{2}\right) = \pi\left(\frac{6}{3} + \frac{1}{3}\right) = \frac{7\pi}{3}$$

$$\Rightarrow P(\ell) = \frac{7\pi}{3}\left(1 + \frac{1}{3}\cos^2 \ell\right)$$