

MOO-2

Dielectric & Image Charge

- a) with an infinite conductor we place an image charge  $q$  at  $-z$  for all time

$$F = -\frac{q^2}{(2z)^2} = -\frac{q^2}{4z^2} = m\ddot{z}$$

$$\text{so } \ddot{z} = -\frac{q^2}{4m} z^{-2}$$

$$\ddot{z}\ddot{z} = -\frac{q^2}{4m} \ddot{z} z^{-2}$$

$$\frac{1}{2} \frac{d}{dt} (\dot{z}^2) = \frac{q^2}{4m} \frac{d}{dt} (z^{-1})$$

$$d(\dot{z}^2) = \frac{q^2}{2m} d(z^{-1})$$

$$\dot{z}^2 = \frac{q^2}{2m} z^{-1} + C$$

we know  $\dot{z}=0$  when  $z=z_0$

$$\text{so } \dot{z}^2 = \frac{q^2}{2mz} - \frac{q^2}{2mz_0}$$

$$\frac{dz}{dt} = \sqrt{\frac{q^2}{2mz} - \frac{q^2}{2mz_0}}$$

$$dt = \frac{dz}{\sqrt{\frac{q^2}{2mz} - \frac{q^2}{2mz_0}}} = \sqrt{\frac{2mz_0}{q^2}} \frac{dz}{\sqrt{\frac{z_0}{z}-1}}$$

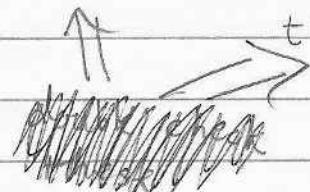
$$\int_0^t dt = \sqrt{\frac{2mz_0}{q^2}} \int_{z_0}^0 \frac{dz}{\sqrt{\frac{z_0}{z}-1}} \quad | \quad u = \frac{z}{z_0}, \quad du = \frac{dz}{z_0}$$

$$\text{let } x = \frac{z_0}{z} \quad dx = -\frac{z_0}{z^2} dz$$

$$z = \frac{z_0}{x} \quad dz = -\frac{z_0}{x^2} dx$$

$$\sqrt{\frac{2m}{q^2}} \int_1^0 \sqrt{\frac{z}{1-\frac{z}{z_0}}} dz$$

$$\text{so } t = \sqrt{\frac{2mz_0^3}{q^2}} \int_{\infty}^1 \frac{dx}{x^2 \sqrt{x-1}}$$



$$t = \sqrt{\frac{2mz_0^3}{q^2}} \int_1^0 \frac{\sqrt{u} du}{\sqrt{1-u}}$$

b) since we neglect relativistic effects, the power radiated is just found from the Larmor formula

$$P = \frac{2e^2 a^2}{3c^3} \quad \text{where } a = 1 \frac{\vec{E}}{m}$$

well as before,  $\vec{F} = -\frac{q^2}{4mz^2} \hat{z}$   
 $\text{so } a = \frac{q^2}{4mz^2}$

so the power radiated by the single point charge is

$$\boxed{P_{\text{real}} = \frac{q^4}{6mz^2 c^3}}$$

the bound charges in the conductor will also radiate; difficult to say how much, but radiation due to image charge might be a good guess (esp. since total bound charge =  $-q$ )

$$P_{\text{image}} = \frac{q^4}{6mz^2 c^3} \quad P_{\text{total}}$$

the total  $P$  would be approximately the radiation due to a dipole moment  $p = 2qz$

$$P = \frac{2p^2}{3c^3} = \frac{8q^2 z^2}{3c^3} \quad \ddot{p} = 2q \ddot{z} = 2qa$$

So total is  $4 \times P_{\text{real}}$

c) for the upper region, take the real charge plus a charge of  $q'$  at  $z=a$

potential is  $V_{up} = \frac{q}{\sqrt{r^2 + (z-z_0)^2}} + \frac{q'}{\sqrt{r^2 + (z+a)^2}}$

for the lower region, again take the real charge, but now an image charge of  $q''$  at  $z+b$

potential is  $V_{low} = \frac{q}{\sqrt{r^2 + (z-z_0)^2}} + \frac{q''}{\sqrt{r^2 + (z-b)^2}}$

now require these potentials to be equal at  $z=0$   
and require their normal derivatives to obey  $\frac{\partial V_{up}}{\partial z} = \epsilon \frac{\partial V_{low}}{\partial z}$

$$V_{up}(0) = V_{low}(0) = \frac{q}{\sqrt{r^2 + z_0^2}} + \frac{q'}{\sqrt{r^2 + a^2}} = \frac{q}{\sqrt{r^2 + z_0^2}} + \frac{q''}{\sqrt{r^2 + b^2}}$$

so let  $q'' = q'$        $b = a$

$$\frac{\partial V_{up}(0)}{\partial z} = + \frac{q}{2(r^2 + z_0^2)^{3/2}} (+2z_0) - \frac{q'}{2(r^2 + a^2)^{3/2}} (2a)$$

$$\frac{\partial V_{low}(0)}{\partial z} = + \frac{q}{2(r^2 + z_0^2)^{3/2}} (-2z_0) - \frac{q'}{2(r^2 + a^2)^{3/2}} (-2a)$$

so  $\frac{q z_0}{(r^2 + z_0^2)^{3/2}} - \frac{q' a}{(r^2 + a^2)^{3/2}} = \epsilon \left( \frac{q z_0}{(r^2 + z_0^2)^{3/2}} + \frac{q' a}{(r^2 + a^2)^{3/2}} \right)$

let  $a = z_0$        $q - q' = \epsilon (q + q')$        $F = \frac{q q'}{(z_0 + a)^2} = \frac{(1-\epsilon)}{(1+\epsilon)} \frac{q^2}{4z_0^2}$

$$(1-\epsilon)q = (1+\epsilon)q'$$

$$q' = \frac{(1-\epsilon)}{(1+\epsilon)} q$$