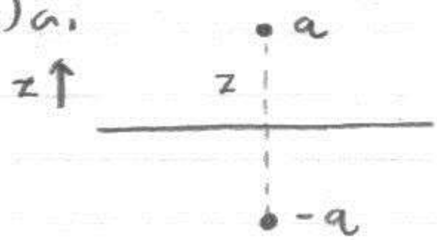
Seth Dorfman Prelims December 22, 2005

May 2000 EM

2) a.



$\vec{F} = \frac{-q^2}{(2z)^2}\hat{z}$

$m\ddot{z} = \frac{-q^2}{4z^2}$

$m\ddot{z}\dot{z} = \frac{-q^2}{4z^2}\dot{z}$

$\frac{\partial}{\partial t}\left(\frac{1}{2}m\dot{z}^2\right) = \frac{\partial}{\partial t}\left(\frac{q^2}{4z}\right)$

$\dot{z}^2 = \frac{1}{2m}\frac{q^2}{z} + C_1$

At $z = z_0$, $\dot{z} = 0$ $\quad \dot{z}^2 = \frac{1}{2m}q^2\left(\frac{1}{z} - \frac{1}{z_0}\right)$

$\frac{dz}{dt} = -q\frac{1}{\sqrt{2m}}\sqrt{\frac{1}{z} - \frac{1}{z_0}}$

$t = \int_{z_0}^{0} -\frac{1}{q}\sqrt{2m}\left(\frac{1}{z} - \frac{1}{z_0}\right)^{-1/2} dz$

Let $x = \frac{z}{z_0}$ $\quad dx = \frac{dz}{z_0}$

$t = \int_1^0 -\frac{1}{q}\sqrt{2m}\left(\frac{1}{xz_0} - \frac{1}{z_0}\right)^{-1/2} z_0\, dx$

$t = \frac{1}{q}\sqrt{2m}\left(\frac{1}{z_0}\right)^{-1/2} z_0 \int_0^1 \left(\frac{1}{x} - 1\right)^{-1/2} dx$

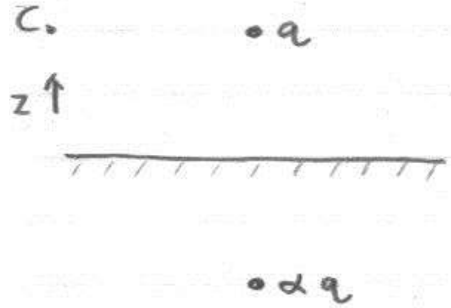
$t = \frac{1}{q}\sqrt{2m}\, z_0^{3/2}\int_0^1 \sqrt{\frac{x}{1-x}}\, dx$

b. $P = \frac{2q^2a^2}{3c^3}$ $\quad \vec{a} = \frac{-q^2}{4mz^2}\hat{z}$

$P = \frac{2q^2}{3c^3}\left(\frac{-q^2}{4mz^2}\right)^2$

$P = \frac{q^6}{24m^2c^3z^4}$

c.



Consider the limiting cases:

1) $\varepsilon = 1$ dielectric $\rightarrow$ vacuum

no image charge $\alpha = 0$

2) $\varepsilon = \infty$ dielectric $\rightarrow$ metal

image charge $-q$ $\alpha = -1$

3) $\varepsilon = 0$ $\quad E^{\perp}_{vac} - \varepsilon E^{\perp}_{diel} = 0$

$\Rightarrow E^{\perp}_{vac} = 0$

requires image charge q $\alpha = 1$

From these cases, guess $\alpha = \frac{1-\varepsilon}{1+\varepsilon}$

$\therefore \vec{F} = \left(\frac{1-\varepsilon}{1+\varepsilon}\right)\frac{q^2}{4z_0^2}\hat{z}$