1 Quantum Mechanics

1.1 Problem 1

1.2 (a)

$$< z\sigma_z >=< g|z\sigma_z|g >$$

where $|g>$ is the ground state of the full Hamiltonian. The perturbation hamiltonian is:

$$H^1 = \lambda \begin{pmatrix} z & x - iy \\ x + iy & -z \end{pmatrix}$$

To first order in λ:

$$|g>= |g^0> +|g^1>= |g^0> + \sum_{m\neq g} \frac{< m^0|H^1|g^0>}{E_g^0 - E_m^0}|m^0>$$

$$< z\sigma_z >=< g^1|z\sigma_z|g^0> + < g^1|z\sigma_z|g^1> + < g^0|z\sigma_z|g^0> + < g^0|z\sigma_z|g^1>$$

$$= \sum_{m\neq g} \frac{< g^0|H^1|m^0>}{E_g^0 - E_m^0} < m^0|z\sigma_z|g^0> + \sum_{m\neq g}\sum_{n\neq g} \frac{< g^0|H^1|m^0>}{E_g^0 - E_m^0} < m^0|z\sigma_z|n^0> \frac{< n^0|H^1|g^0>}{E_g^0 - E_n^0}$$

$$+ < g^0|z\sigma_z|g^0> + \sum_{m\neq g} < g^0|z\sigma_z|m^0> \frac{< m^0|H^1|g^0>}{E_g^0 - E_m^0}$$

The second term is second order in λ so we can throw it out. Then we get:

$$< z\sigma_z >=< g^0|z\sigma_z|g^0> + \sum_{m\neq g} \frac{H^1_{gm} < m^0|z\sigma_z|g^0> + H^1_{mg} < g^0|z\sigma_z|m^0>}{E_g^0 - E_m^0}$$

where $H^1_{ab} =< a^0|H^1|b^0>$. We can express z in terms of raising and lowering operators:

$$z = \frac{a_z^+ + a_z}{\sqrt{2m\omega}}$$

$$< m^0|z\sigma_z|g^0>= \sigma_z < m^0|z|g^0>= \frac{\sigma_z}{\sqrt{2m\omega}} < m^0|(a_z^+ + a_z)|g^0>= \frac{\sigma_z}{\sqrt{2m\omega}} < m^0|1_z^0>= \frac{\sigma_z}{\sqrt{2m\omega}}\delta_{m1_z}$$

$$< z\sigma_z >= \sum_{m\neq g} \frac{H^1_{gm}\frac{\sigma_z}{\sqrt{2m\omega}}\delta_{m1_z} + H^1_{mg}\frac{\sigma_z}{\sqrt{2m\omega}}\delta_{m1_z}}{E_g^0 - E_m^0} = \frac{H^1_{g1_z}\frac{\sigma_z}{\sqrt{2m\omega}} + H^1_{1_z g}\frac{\sigma_z}{\sqrt{2m\omega}}}{E_g^0 - E_{1_z}^0}$$

$$H^1_{g1_z} =< g^0|H^1|1_z^0>= \lambda\sigma_z < g^0|z|1_z^0>= \lambda\frac{\sigma_z}{\sqrt{2m\omega}}$$

$$< z\sigma_z >= \frac{\lambda\frac{\sigma_z^2}{2m\omega} + \lambda\frac{\sigma_z^2}{2m\omega}}{\omega/2 - 3\omega/2} = \frac{2\lambda\frac{I_2}{2m\omega}}{-\omega} = -\frac{\lambda}{m\omega^2}I_2$$

where I_2 denotes the the 2×2 identity matrix.