J98M.3

Solution to J98M.3 — Ice Skate

a) The equation of motion can be derived straightly from Newton's law. The gravity provides a uniform force on x direction, while the normal force can also be decomposed into x and y direction. Thus, we have,

$$\begin{aligned} m\ddot{x} &= \frac{1}{2}mg - N\sin(\phi) \\ m\ddot{y} &= N\cos(\phi) \end{aligned} \tag{1}$$

b) Since the blade cannot move perpendicular to itself, the velocity always points at $\tan(\phi)$ direction. So the equation of constrain is

$$\frac{\dot{y}}{\dot{x}} = \tan(\phi) \tag{2}$$

c) The normal force is perpendicular to the blade at all time. So $N = Kmg\sin(\phi)$, where K is a constant. Also, the Lagrangian $L = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) + \frac{1}{2}I\dot{\phi}^2 - V(x)$ implies $p_\phi = const$, which means $\phi(t) = \omega t$. After I get rid of N, the new equation of motion is

$$\begin{aligned} \ddot{x} &= \frac{1}{2}g - Kg\sin^2(\omega t) \\ \ddot{y} &= Kg\sin(\omega t)\cos(\omega t) \end{aligned} \tag{3}$$

Integrate both side over t and apply initial conditions. Now we have,

$$\dot{x} = \frac{1}{2}gt - \frac{Kg}{2}t + \frac{Kg}{4\omega}\sin(2\omega t) \tag{4}$$
$$\dot{y} = \frac{Kg}{4\omega}\left[1 - \cos(2\omega t)\right]$$

Notice, the constrain equation 2 is satisfied only if K=1. Thus, we are able to fix K and continue to find the motions.

$$x(t) = \frac{g}{8\omega^2}\left[1 - \cos(2\omega t)\right] \tag{5}$$
$$y(t) = \frac{g}{4\omega}t - \frac{g}{8\omega^2}\sin(2\omega t)$$

This is a Cycloid.

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2 thoughts on “J98M.3”

October 1, 2013 at 2:28 pm

Much better now. The new solution is correct.
You significantly simplified it by guessing the right ansatz for N (with K being constant).

You probably realize it, but if not, I want to emphasize that you could solve the problem without guessing the right expression for N just by solving the system of equations of motion and constraints (1) and (2) (that's what you do in a general constrained system). In such a case you would derive the same expression for N by solving some differential equation. However, since you were able to guess the ansatz and show at the end that the constraint is satisfied, you avoided that differential equation.

September 29, 2013 at 8:58 pm

You're on the right track. However, the solution is not correct so far.
Notice that if you substitute equations (4) into the constraint $\frac{\dot{y}}{\dot{x}} = \tan(\phi)$, the constraint won't be satisfied. What's wrong?

Hint: your expression for N implies that normal acceleration (not velocity) is zero. Think why vanishing normal acceleration doesn't imply vanishing normal velocity in our case.