~~m98 j~~ j98 m1

a. minimize $-\rho g \int_{-w}^{w} y\sqrt{1+y'^2}\,dx = -\rho g \int_{-w}^{w} \underbrace{y\sqrt{1+x'^2}}^{\mathcal{L}}\,dy$

such that $\int_{-w}^{w} \sqrt{1+y'^2}\,dx = 2L$.

b. $\frac{d}{dy}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}}\right) = 0 \rightarrow \frac{\partial \mathcal{L}}{\partial \dot{x}} = \frac{yx'}{\sqrt{1+x'^2}}$ is conserved.

c. $\frac{yx'}{\sqrt{1+x'^2}} = A$

$y^2 \dot{x}^2 = A^2 + A^2\dot{x}^2$

$\dot{x}^2(y^2 - A^2) = A^2 \rightarrow \dot{x} = \frac{A}{\sqrt{y^2-A^2}}$

$y = A\cosh\left(\frac{x}{A}\right)$ works.

constraint: $\int_{-w}^{w} \sqrt{1+y'^2}\,dx = 2L$

$\cosh^2 - \sinh^2 = 1$

$2\int_0^w \sqrt{\cancel{\sinh^2}\cosh^2\left(\frac{x}{A}\right)}\,dx = 2L$

$\boxed{A\sinh\left(\frac{w}{A}\right) = L}$ determines A.

$y = A\cosh\left(\frac{x}{A}\right)$.

curiously: independent of ρ/M and g!