We set $R = a$

Laplace's equation: $\nabla^2 V = 0$

$$V(r, \theta) = \sum_{l=0}^{\infty} (A_l r^l + B_l/r^{l+1}) P_l(\cos(\theta))$$

For $r > R$:

Boundary condition: $r >> R$, $V = -E_0 r \cos(\theta)$ so only $A_1$ survives (or $l = 1$) out of all the $A_l$'s.

Boundary condition: $r = R$, $V = 0$.

Using these boundary conditions and the linear independence of $P_l$ we obtain:

$$V(r > R, \theta) = (A_1 r + B_1/r^2)\cos(\theta) = E_0 \cos(\theta)(-r + R^3/r^2)$$

We find the electric field:

$$E(r > R, \theta) = -\nabla V = (E_0 + 2E_0 R^3/r^3)\cos(\theta)\hat{r} + (-E_0 + E_0 R^3/r^3)\sin(\theta)\hat{\theta}$$

At $r = R$:

$$E(R^+, \theta) = 3E_0 \cos(\theta)\hat{r}$$

Inside the conductor $E = 0$: 
We find the surface charge: \( \sigma = \varepsilon_0 (E_{\text{out}} - E_{\text{in}}) = 3\varepsilon_0 E_0 \cos(\theta) \)

The surface charge generates an \( E_{\text{gamma}} = \sigma/(2\varepsilon_0) = (3/2)E_0 \cos(\theta)\hat{r} = E_{\text{ext}} \) on each side

So we may now find the differential force: \( dF = E_{\text{ext}} dq = E_{\text{ext}} \sigma(\theta) dA = (9/2)\varepsilon_0 E_0^2 \cos^2(\theta) dA \hat{r} \)

And so the force (on either top or bottom hemispheres):

(we replace \( \hat{r} \) with \( \cos(\theta)\hat{z} \) because the \( x \) and \( y \) components of the force on one hemisphere cancel out due to symmetry around \( \phi \))

\[
F = \int_0^{2\pi} \int_0^{\pi/2} d\theta d\phi R^2 \sin(\theta) (9/2)\varepsilon_0 E_0^2 \cos^2(\theta) (\cos(\theta)\hat{z})
\]

\[
F = 9\pi R^2 \varepsilon_0 E_0^2 \int_0^{\pi/2} \sin(\theta) \cos^3(\theta) d\theta
\]

\[
F = - (9/4)\pi R^2 \varepsilon_0 E_0^2 \cos^4(\theta) \bigg|_0^{\pi/2} = (9/4)\pi R^2 \varepsilon_0 E_0^2
\]

This force is required on each hemisphere to prevent them from separating.

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**One thought on “J98E.3”**

December 8, 2013 at 11:37 pm

Everything looks correct.

Perfect, except that you didn't tell that \( B_0 = 0 \) was due to the vanishing total charge.