January 2018 Princeton Physics Preliminary Exam Notes

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Disclaimer:

I did not take the exam or any notes on the questions with me from the testing location. I wrote these notes within a few hours of taking each day's exam, which really helped me recall the questions in detail. The questions were much better worded than my notes here (I thought they were especially well-defined this year), but I think I have given at least the necessary details for solving these problems. Good luck!

Problems:

J18M.1: Consider a potential $V(x,y) = \frac{1}{2}\alpha(x^2 - y^2)$ rotating at an angular frequency ω, which can stabilize an ion in this rotating "trap" when non-inertial forces are considered. There was a long back story on trying to trap ions with an electrostatic potential, and why it doesn't work (has to do with the characteristics of solutions to Laplace's equation). The story is unnecessary to solving the problem. The ion's mass and charge are not given.

a) Find equation of motion in the x-y plane.
b) Find algebraic expression that classifies possible small oscillation frequencies (ω is complex)
c) Find all rotation frequencies Ω such that ω is real.

J18M.2: Waves on a string with a point mass attached. I didn't do this problem, so I don't remember any of the parts.

J18M.3: Solid sphere of mass M and radius R rotates with constant angular velocity ω_0 about the z axis. A small mass with zero angular momentum, mass αM, collides with the sphere at some polar angle θ and sticks to the sphere's surface. The moment of inertial of the sphere is $I = \frac{2}{5}MR^2$

a) Find principle axes and moment of inertia tensor in this basis.
b) Find the period of free precession in terms of the original frequency ω_0.

J18E.1: Explore Faraday effect, where a semi-infinite medium has different indices of refraction ($n+$ or $n-$) depending on if the wave is right or left circularly polarized. Incident from vacuum on this semi-infinite medium is a linearly polarized wave, polarized in the perpendicular direction. (There was a diagram to go with the description)

a) Find the ratio of the reflected wave intensity to the incident wave intensity.
b) Qualitatively describe the polarization of the reflected wave.

J18E.2: Image charge outside a conducting sphere (Extremely similar to parts (a) - (d) of M03E.2)

a) For a charge Q at a distance R from the center of a sphere of radius a, what is the image charge magnitude and it's position. Do not need to prove $V = 0$ on the surface of the sphere.
b) What is the force on the sphere of radius a if a point charge Q is a distance R from the center of this sphere and the sphere has total charge Q?
c) Find an equation that describes the farthest radius R for the point charge Q such that there is a point on the sphere with zero surface charge density.
d) Consider two conducting spheres of radius a separated by a distance $R \gg 2a$. These spheres are held at potential V_0 by a thin wire. This would normally result in an infinite series of image charges. Find the total charge on one of the spheres to leading order in (a/R).

J18E.3: Given the Maxwell stress-energy tensor expression. Told that the electromagnetic force experienced by charges is found using the appropriate surface integral of the appropriate components of this tensor. Be specific about the surface across which the integral is taken and the sign of the integral.

a) Find the force per unit area on the parallel plates of a capacitor, separated by a distance $2a$ with surface charge densities $+\sigma$ and $-\sigma$.

b) Find the force per unit length between two line charges of linear charge density $+\lambda$ and $-\lambda$.

c) Find the force per unit length between two line charges of the same linear charge density $+\lambda$.

J18Q.1: Extremely similar to M01Q.2 (Asked a lot of the same questions) I didn't do this problem, so I don't really have any more details than that. May have been a complete repeat of the question.

J18Q.2: Consider a potential barrier that linear drops from $V(x=0)=V_1$ to $V(x=a)=V_2$, $V_1 > V_2$. Let the wave function for $x<0$ be $\psi_L = e^{ikx} + Re^{-ikx}$. Let the wave function for $x>a$ be $\psi_R = Te^{ikx}$. We will use the WKB approximation to estimate tunneling through this barrier for a plane wave with energy $E < V_2 < V_1$. (There was a picture to go with the description)

a) Write down the WKB wave functions in the barrier region. Remember that there are two wave functions, which you'll need to describe using two separate, possibly complex amplitudes.

b) Show that the probability flux through the barrier is independent of x. Remember, assume the amplitudes to be complex. You can also use the WKB assumption that $\frac{d}{dx}(\log K(x)) << K(x)$, where $K(x) = \sqrt{2m(V(x)-E)}$. If it isn't revisit your wave functions in part (a).

c) The WKB wave function in the barrier has to match up with the wave functions ψ_L and ψ_R at $x=0$ and $x=a$, respectively. Write down two sets of conditions to make sure this happens. No need to solve right now.

d) Assume that the barrier is extremely wide (large a). Use this to simplify your conditions from part (c), along with the WKB assumptions to estimate the transmission coefficient T to first non-vanishing order.

J18Q.3: Model a white dwarf star as a degenerate Fermi gas of N protons and N electrons. The star is approximately a sphere of radius R. The electrons support the star from gravitational collapse by their electron degeneracy pressure contribution. For simplicity, assume uniform mass density, not kinetic considerations of the protons, and no mass contribution from the electrons when calculating the gravitational potential energy -> Mass of star $M = Nm_p$. (Very similar to M03T.2, with less given information. Also, that's not a typo. Day 2 of J18 prelims consisted of 2 quantum and 4 stat mech questions)

a) Assume the electrons are non-relativistic. Find their Fermi energy and the kinetic energy of the electron gas. (M03T.2 gives the kinetic energy, which you must verify. We were not given this info)

b) The gravitation binding energy of a uniform density sphere is

$$U_{grav} = -\frac{3GM^2}{5R} \tag{1}$$

Find the equilibrium radius for the white dwarf. How does it depend on the mass? At what mass do the electrons become relativistic?

c) Now, assume the electrons are highly relativistic. Calculate the Fermi energy and the kinetic energy of the electron gas. (Again, M03T.2 gives the kinetic energy to verify. We were not.)

d) Under what conditions is this highly relativistic degenerate electron start stable? This is called the Chandrasekhar limit. Stars that violate the limit will collapse into a neutron star or black hole.

J18T.1: Particle in a Box. Repeated question M03T.3. No modifications to the problem were made.

J18T.2: Steven Hawking proposed that black holes can be thought of using thermodynamic principles. He proposed the thermodynamic energy of a black hole to be Mc^2, where M is the mass of the black hole. He also proposed that the entropy of a black hole would be proportional to $\frac{1}{4}$th of the area of the black hole's event horizon, as follows:

$$\frac{S}{k_B} = \frac{CM^2}{4G\hbar/c^3} \tag{2}$$

where C represents some constants that I don't remember (They don't really matter for the problem.)

a) One can relate the temperature of a thermodynamic system to the entropy using an appropriate derivative. State this relationship and use it to derive the thermodynamics "temperature" of a black hole.

b) Assume black holes radiate as black bodies of this temperature, found in part (a), radiating photons from their surface. Find the rate of power loss due to this radiation.

c) Assuming the rate of power loss you found in part (b), calculate the time a black hole of mass M would take to completely dissipate.

J18T.3: Particles on a Line. Repeated question J94T.2, M06T.1, and M09T.2 (Same Hamiltonian and questions) The only difference was an additional question to previous appearances, asking about intermediate temperature behavior of this system.