Problem J18E.1

For a right-circularly polarized incident beam, the incident E-field satisfies

$$\mathbf{E}_{i,+}(z,t) = E_0 \begin{bmatrix} \cos(\omega t - kz) \\ \sin(\omega t - kz) \\ 0 \end{bmatrix},$$

and so the reflected E-field satisfies

$$\mathbf{E}_{r,+}(z,t) = E_0 \frac{1-n_+}{1+n_+} \begin{bmatrix} \cos(\omega t + kz) \\ \sin(\omega t + kz) \\ 0 \end{bmatrix}.$$

A similar identity holds for left-circularly polarized beam. The polarization of the incident linearly polarized beam can be written as

$$\mathbf{E}_i(z,t) = E_0 \begin{bmatrix} \cos(\omega t - kz) \\ 0 \\ 0 \end{bmatrix} = \frac{E_0}{2} \begin{bmatrix} \cos(\omega t - kz) \\ \sin(\omega t - kz) \\ 0 \end{bmatrix} + \frac{E_0}{2} \begin{bmatrix} \cos(\omega t - kz) \\ -\sin(\omega t - kz) \\ 0 \end{bmatrix},$$

so the reflected beam has the polarization

$$\begin{aligned} \mathbf{E}_r(z,t) &= \frac{E_0}{2} \begin{bmatrix} \left(\frac{1-n_+}{1+n_+} + \frac{1-n_-}{1+n_-}\right) \cos(\omega t - kz) \\ \left(\frac{1-n_+}{1+n_+} - \frac{1-n_-}{1+n_-}\right) \sin(\omega t - kz) \\ 0 \end{bmatrix} \\ &= \frac{E_0}{(1+n_+)(1+n_-)} \begin{bmatrix} (1-n_+n_-)\cos(\omega t - kz) \\ (n_- - n_+)\sin(\omega t - kz) \\ 0 \end{bmatrix}. \end{aligned}$$

Thus the reflected beam is elliptically polarized. The intensity of the incident beam is proportional to

$$\langle \mathbf{E}_i^2 \rangle = \frac{1}{2} E_0^2,$$

and the intensity of the reflected beam is proportional to

$$\langle \mathbf{E}_r^2 \rangle = \frac{1}{2} E_0^2 \frac{(1-n_+n_-)^2 + (n_- - n_+)^2}{(1+n_+)^2(1+n_-)^2}$$

so that

$$\frac{I_r}{I_i} = \frac{(1-n_+n_-)^2 + (n_- - n_+)^2}{(1+n_+)^2(1+n_-)^2}.$$

As expected, when $n_+ = n_- = n$, this expression reduces to $(1-n)^2/(1+n)^2$.