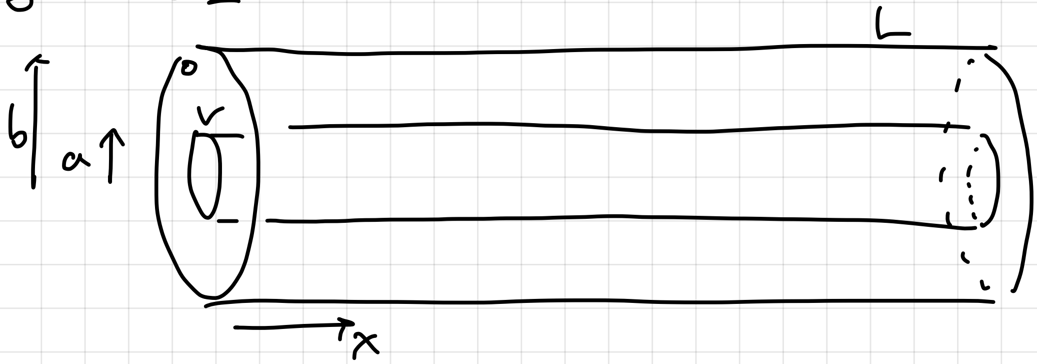
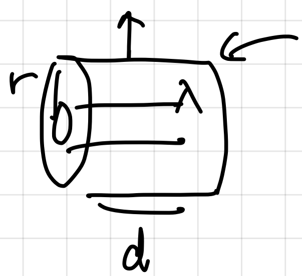
Jin E2



(a) $V(t): 0 \to V_0$ $\qquad \vec{S}$?

$\boxed{\text{Step 1}}$ Find $\vec{E}$



cylinder gauss surface with radius r.

$$2\pi r d\, E = \frac{\lambda d}{\varepsilon_0} \qquad E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}.$$

$$\int_a^b \vec{E}\cdot d\vec{r} = \int_a^b \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}dr = \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{b}{a} = V$$

$$\therefore \lambda = \frac{2\pi\varepsilon_0 V}{\ln(b/a)} \qquad \vec{E}(r,t) = \frac{1}{\ln(b/a)}\frac{V(t)}{r}\hat{r}.$$

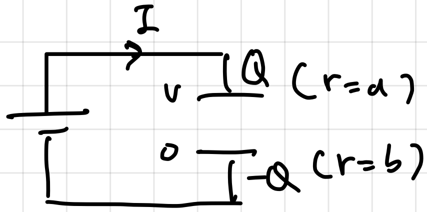
$\boxed{\text{Step 2}}$ Find $I(t)$

Co-axial cylinder works as a capacitor.

$$Q = CV$$

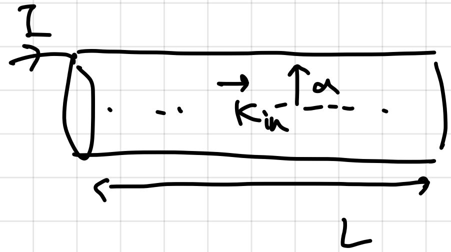
calculated from Step 1

$$\lambda L = C\cdot \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{b}{a} \Rightarrow C = \frac{2\pi\varepsilon_0 L}{\ln(b/a)}$$



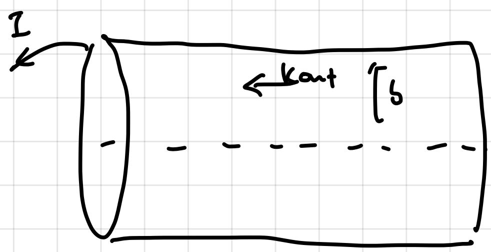
$$V - \frac{Q}{C} = 0$$

$$\therefore I = \frac{dQ}{dt} = \frac{d}{dt}(CV) = C\dot{V}$$



$$2\pi a K_{in}(x) = \frac{d}{dt} \cdot Q \cdot \frac{L-x}{L}$$

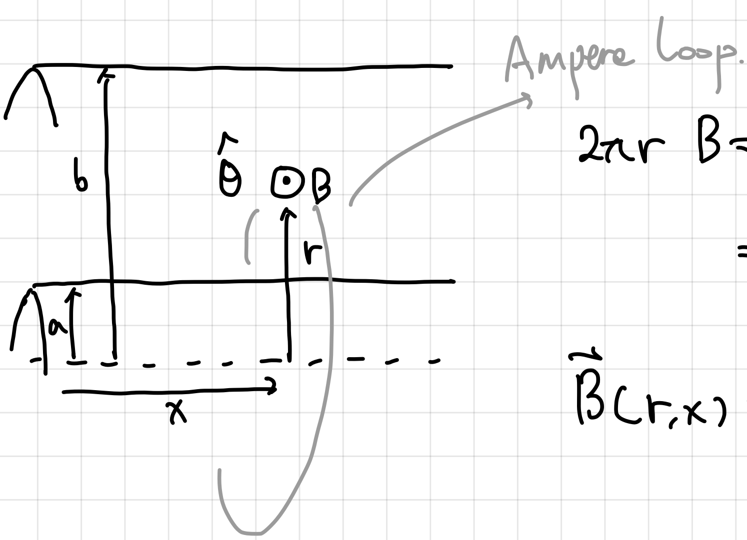
$$K_{in}(x) = \frac{L-x}{2\pi a L} C\dot{V}$$



$$2\pi b K_{out}(x) = -\frac{d}{dt} Q \frac{L-x}{L}$$

$$K_{out}(x) = \frac{L-x}{2\pi b L} C\dot{V}$$

$\boxed{\text{Step 3}}$ Find tangential B.



$$2\pi r B = \mu_0 I(x) = \mu_0 \cdot 2\pi a K_{in}(x)$$

$$= \mu_0 \frac{L-x}{L} C\dot{V}$$

$$\vec{B}(r,x) = \frac{\mu_0}{2\pi} \frac{L-x}{L} \frac{C\dot{V}}{r} \hat{\theta}$$

$$= \frac{\varepsilon_0 \mu_0}{\ln(b/a)} \frac{L-x}{r} \dot{V} \hat{\theta}$$

$$= \frac{1}{c^2} \frac{1}{\ln(b/a)} \frac{L-x}{r} \dot{V} \hat{\theta}$$

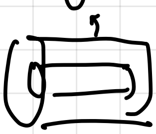
$\boxed{\text{Step4}}$ Find $\vec{S}(r,x)$

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$$

$$= \frac{1}{\mu_0}\left(\frac{1}{\ln(b/a)}\frac{V}{r}\hat{r}\right)\times\left(\frac{1}{c^2}\frac{1}{\ln(b/a)}\frac{L-x}{r}\dot{V}\hat{\theta}\right)$$

$$= \varepsilon_0\frac{1}{\ln(b/a)^2}\frac{L-x}{r^2}V\dot{V}\;\hat{x}$$

$$\frac{dE}{dt} = \int\vec{S}\cdot d\vec{a} = -\int_a^b \varepsilon_0\frac{1}{\ln(b/a)^2}\frac{L}{r^2}V\dot{V}\cdot 2\pi r\,dr$$ (only $x=0$ contributes)

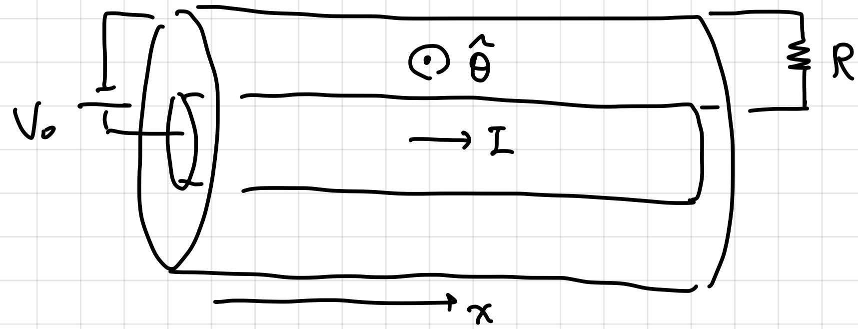


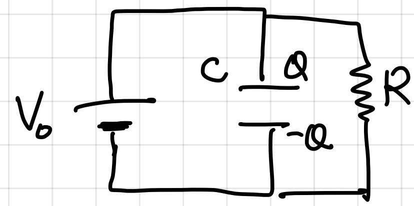
$$= -\varepsilon_0\frac{2\pi L}{\ln(b/a)^2}V\dot{V}\ln(b/a) = -\frac{2\pi\varepsilon_0 L}{\ln(b/a)}V\dot{V} = -\frac{\pi\varepsilon_0 L}{\ln(b/a)}\frac{d(V^2)}{dt}$$

$$\Delta E = \int_0^{t\to\infty}\frac{dE}{dt} = -\frac{\pi\varepsilon_0 L}{\ln(b/a)}\left(0-V_0^2\right) = \frac{\pi\varepsilon_0 L}{\ln(b/a)}V_0^2.$$

As $C = \frac{2\pi\varepsilon_0 L}{\ln(b/a)}$, $\Delta E = \frac{1}{2}CV_0^2.$

(b)





$I = \frac{V_o}{R}$: constant current flows along the cable.

$\therefore\ \vec{B}(r,x) = \frac{\mu_0 I}{2\pi r}\hat{\theta}$

As calculated in $\vec{E}$,

$\vec{E}(r,x) = \frac{1}{\ln(b/a)}\frac{V_o}{r}\hat{r}$

$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} = \frac{V_o^2}{2\pi R}\frac{1}{\ln(b/a)}\frac{1}{r^2}\hat{x}$

$\vec{p} = \frac{1}{c^2}\int_V \vec{S}\cdot d\tau = \frac{L}{c^2}\int_a^b S\cdot 2\pi r\, dr\ \hat{x}$

$= \frac{L}{c^2}\int_a^b \frac{V_0^2}{2\pi R}\frac{1}{\ln(b/a)}\frac{2\pi r}{r^2}dr\,\hat{x} = \frac{L}{c^2}\cdot\frac{V_0^2}{R}\hat{x}$

$\boxed{\vec{p} = \frac{L}{c^2}\frac{V_0^2}{R}\hat{x}}$

momentum can be stored in E&M field.
when $I \to 0$, the momentum in the field will transform to the mechanical momentum.