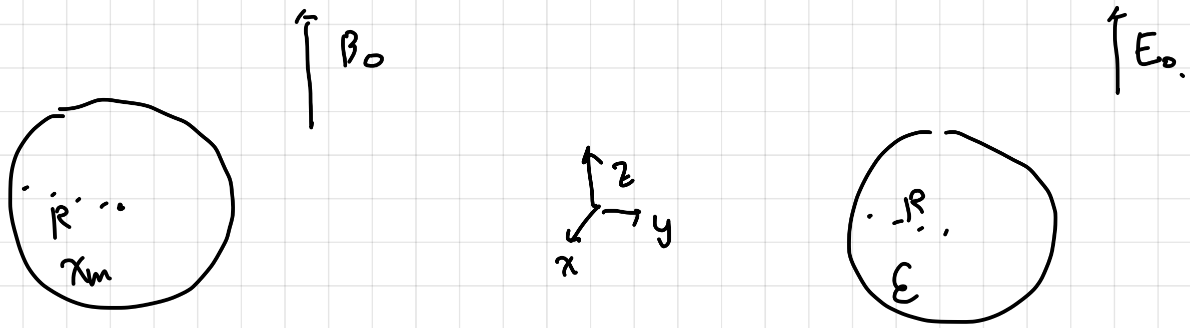
J 17. E1



Note that it is LINEAR diamagnetic material.

inside / outside. of the sphere. $\vec{H}$ field should piece-wisely satisfy

- $\nabla \cdot \vec{H} = \frac{1}{\mu}(\nabla \cdot \vec{B}) = 0$
- $\nabla \times \vec{H} = \mu_0 J_f = 0.$

B.C.

$r \to \infty \qquad \vec{H} = \frac{1}{\mu_0} B_0 \hat{z}$

$r = a \qquad \nabla \cdot \vec{B} = 0 : \qquad \hat{r} \cdot \left(\frac{1}{\mu_0} \vec{H}_{out}\right) = \hat{r} \cdot \left(\frac{1}{\mu} \vec{H}_{in}\right).$

We know the solution for this problem.

Think of a sphere with ε in uniform $E_0 \hat{z}$ background field

$\nabla \cdot \vec{E} = 0$ (we know the answer. Fortunately, this holds)

$\nabla \times \vec{E} = 0.$

B.C.

$r \to \infty \qquad \vec{E} = E_0 \hat{z}$

$r = a \qquad \nabla \cdot \vec{D} = 0 : \quad \hat{r} \cdot (\varepsilon_0 \vec{E}_{out}) = \hat{r} \cdot (\varepsilon \vec{E}_{in})$

So we change

$E_0 \Leftrightarrow \frac{1}{\mu_0} B_0$

$\varepsilon_0 \Leftrightarrow \frac{1}{\mu_0}$

$\varepsilon \Leftrightarrow \frac{1}{\mu} = \frac{1}{\mu_0(1+\chi_m)}$.

$B = \mu H = \mu_0(H+M) = \mu_0(1+\chi_m)H$

$\therefore \mu = \mu_0(1+\chi_m)$

It is easy to find the answer for $\vec{E}$ fields.

(Use $V(r) = \sum_\ell \left(A_\ell r^\ell + \frac{B_\ell}{r^{\ell+1}}\right) P_\ell(\cos\theta)$ and impose boundary condition...)

I'll just write the answer. $\vec{E}_{in} = \frac{3\varepsilon_0}{\varepsilon + 2\varepsilon_0} \vec{E_0}$

(See Griffith E&M for details)

$$\therefore \vec{H}_{in} = \frac{\frac{3}{\mu_0}}{\frac{1}{\mu_0(1+\chi_m)} + \frac{2}{\mu_0}} \frac{\vec{B_0}}{\mu_0} = \frac{3(1+\chi_m)}{3+2\chi_m} \frac{\vec{B_0}}{\mu_0}.$$

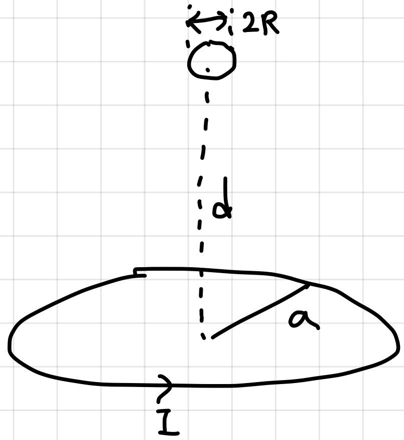
$$\vec{M} = \chi_m \vec{H} \quad : \quad \vec{m} = \frac{4}{3}\pi R^3 \cdot \vec{M} = \boxed{4\pi R^3 \cdot \frac{\chi_m(1+\chi_m)}{3+2\chi_m} \cdot \frac{\vec{B_0}}{\mu_0}}$$

$-1 < \chi_m < 0 \qquad \vec{m} \parallel -\vec{B_0}$

$-3/2 < \chi_m < -1 \qquad \vec{m} \parallel \vec{B_0}$

$\chi_m < -3/2 \qquad \vec{m} \parallel -\vec{B_0}$

(b)



$$\vec{F} = \nabla(\vec{m}\cdot\vec{B})$$

Using Biot-Savart Law (on the axis)

$$\vec{B}(d) = \frac{\mu_0}{4\pi}\int \frac{I d\vec{l}\times\hat{r}}{r^2} = \frac{\mu_0}{4\pi}\cdot 2\pi a \cdot \frac{aI}{(a^2+d^2)^{3/2}}\,\hat{z}$$

$$= \frac{\mu_0 a^2 I}{2}(a^2+d^2)^{-3/2}\,\hat{z}.$$

$$\vec{m} = \alpha\vec{B} \quad \left(\alpha = \frac{4\pi R^3}{\mu_0}\frac{\chi_m(1+\chi_m)}{3+2\chi_m}\right)$$

$$\vec{m}\cdot\vec{B} = \alpha\vec{B}\cdot\vec{B} = \alpha B^2 = \alpha\left(\frac{\mu_0 a^2 I}{2}\right)^2(a^2+d^2)^{-3}.$$

$$\nabla(\vec{m}\cdot\vec{B}) = \frac{\partial}{\partial d}(\vec{m}\cdot\vec{B})\,\hat{d} = \alpha\left(\frac{\mu_0 a^2 I}{2}\right)^2(-3)(a^2+d^2)^{-4}\,2d$$

$$\therefore \vec{F} = \frac{4\pi R^3}{\mu_0}\frac{\chi_m(1+\chi_m)}{3+2\chi_m}\left(\frac{\mu_0 a^2 I}{2}\right)^2(-3)\cdot 2d\cdot\frac{1}{(a^2+d^2)^4}$$

$$= -6\pi R^3\mu_0\frac{\chi_m(1+\chi_m)}{3+2\chi_m}\frac{a^4 d}{(a^2+d^2)^4}I^2.$$

if $\frac{\chi_m(1+\chi_m)}{3+2\chi_m} < 0$, $\vec{m}$ gets levitating force.