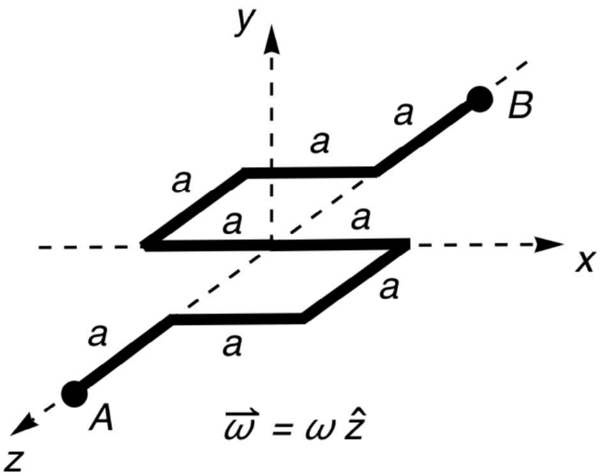
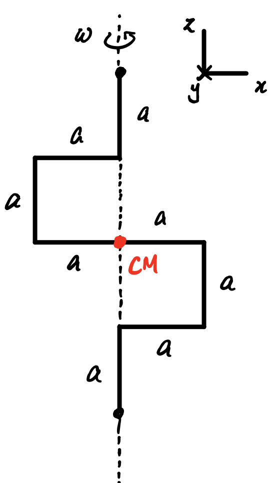
1. **Rotating Crankshaft**

An automobile crankshaft is a planar rigid body made of 8 rods each of mass m, length a, welded together as shown. Suppose the crankshaft rotates about the z axis with constant angular velocity $\omega > 0$. Find the directions and magnitudes of the forces on the two bearings A and B at a moment when the crankshaft lies in the $x-z$ plane as shown. The bearings are located on the ends of the two rods which lie along the x axis. Ignore gravity.





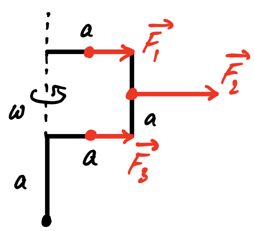
Assume ē CM is at ē origin.

ē off-axis components experience (in ē rotat^g frame) an outward centrifugal force.

These produce a nonzero torque about ē y-axis through ē CM.

ē bear^{g}s counter ē torque by exert^g an oppos^g torque.

Consider:



$\vec{F_2} = ma\omega^2$

$$\vec{F_1} = \vec{F_3} = \int dm\,\omega^2 r$$

$$= \int_0^a dr\,\frac{m}{a}\omega^2 r$$

$$= \frac{m\omega^2}{2a}\cdot a^2 = \frac{1}{2}ma\omega^2$$

ē torque on ē CM is then: $\vec{\tau}_{tot} = \left(|\vec{F_3}|\cdot 2a + |\vec{F_2}|\cdot a\right)\hat{y}$

$$= 2ma^2\omega^2$$

Thus, each bear^g must exert an oppos^g force of $|\vec{F}| = \frac{|\vec{\tau}|}{4a} = \frac{1}{2}ma\omega^2$, w/ $\vec{F_A} = \frac{1}{2}ma\omega^2(-\hat{x})$, $\vec{F_B} = \frac{1}{2}ma\omega^2\hat{x}$.