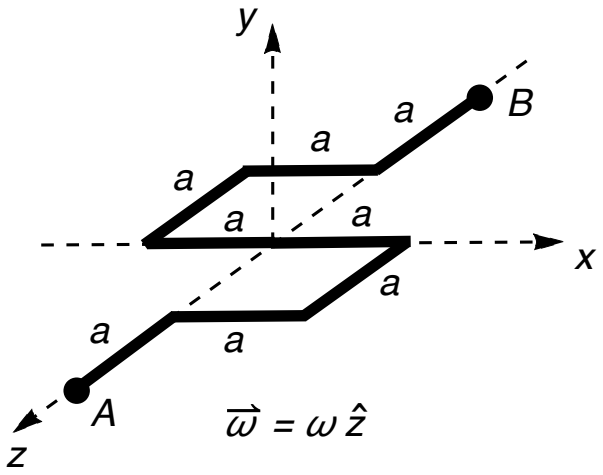
21 Dec 2021

Section A. Mechanics

1. **Rotating Crankshaft**

   An automobile crankshaft is a planar rigid body made of 8 rods each of mass m, length a, welded together as shown. Suppose the crankshaft rotates about the z axis with constant angular velocity $\omega > 0$. Find the directions and magnitudes of the forces on the two bearings A and B at a moment when the crankshaft lies in the $x - z$ plane as shown. The bearings are located on the ends of the two rods which lie along the x axis. Ignore gravity.



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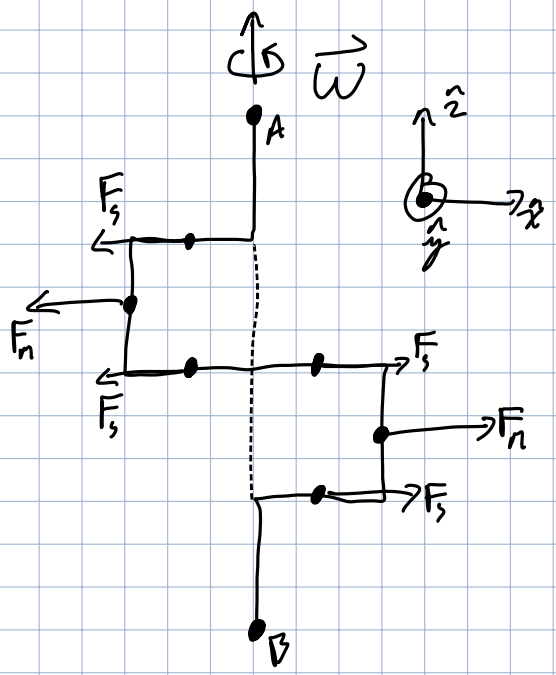

In the stationary frame of the shaft, the rotation exacts centrifugal "forces" at the center of masses of each rod with respects to the axis of rotation

$F_n = \frac{mv^2}{r} = mr\omega^2 = ma\omega^2$, $F_s = mr\omega^2 = \frac{1}{2}ma\omega^2$

$\sum \tau_{\text{about center}} = (a \cdot F_s) + (\frac{a}{2} \cdot F_n) + (0 \cdot F_s) + (0 \cdot F_s) + (\frac{a}{2} \cdot F_n) + (a \cdot F_s)$

$= 2aF_s + aF_n = ma^2\omega^2 + ma^2\omega^2 = 2ma^2\omega^2$

$\sum \tau_{\text{system}} = 0 = 4aF_{A/B} - 2ma^2\omega^2 \rightarrow F_{A/B} = \frac{1}{2}ma\omega^2$

Induced torque: $\hat{\omega}_I = \hat{y} \rightarrow \hat{F}_A = -\hat{x}$; $\hat{F}_B = \hat{x}$

$$\vec{F}_A = -\frac{1}{2}ma\omega^2\hat{x}$$

$$\vec{F}_B = \frac{1}{2}ma\omega^2\hat{x}$$