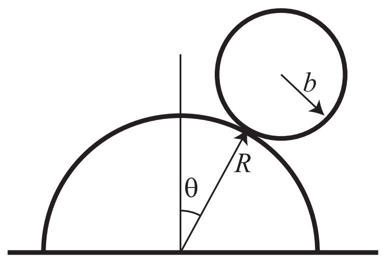
4 Dec 2021

3. A uniform cylinder of mass m and radius b rolls off a fixed cylindrical surface of radius R under the influence of gravity. The axes of both cylinders are horizontal. The rolling cylinder starts from the top of the fixed cylinder with a negligibly small velocity.



(a) If we assume the cylinder rolls without slipping, find the angle θ from the vertical when it looses contact with the fixed cylinder.

(b) In practice for a finite value of μ the cylinder will start to slip before it looses contact. Find the angle when it starts to slip for $\mu = 1$.

JISM.3

forces in $\hat{r}$: $ma_r = F_N - mg\cos\theta \equiv F_c = -\frac{mv^2}{(R+b)}$

forces in $\hat{\theta}$: $ma_\theta = mg\sin\theta - F_f \to$ no slip: $a_\theta = b\alpha = \left(b\left(\frac{F_f b}{I}\right)\right) = b^2 \frac{2}{mb^2} F_f = \frac{2}{m} F_f$

a) $F_N = 0$ ← not pressing on dome

$mg\cos\theta = \frac{mv^2}{(R+b)} \to \cos\theta = \frac{1}{g(R+b)} v^2$

get v^2 from energy conservation

$U_i = mg(R+b)$, $U_f = mg(R+b)\cos\theta$, $T_i = 0$, $T_f = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$; no slip $\to v = b\omega$

$$mg(R+b) = mg(R+b)\cos\theta + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mg(R+b)\cos\theta + \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mb^2\right)\left(\frac{v}{b}\right)^2$$

$$= mg(R+b)\cos\theta + \frac{1}{2}mv^2 + \frac{1}{4}mv^2$$

$$= mg(R+b)\cos\theta + \frac{3}{4}mv^2$$

$$v^2 = \frac{4}{3}g(R+b)(1-\cos\theta)$$

↓

$$\cos\theta = \frac{4}{3}(1-\cos\theta) = \frac{4}{3} - \frac{4}{3}\cos\theta$$

$$\frac{7}{3}\cos\theta = \frac{4}{3} \to \cos\theta = \frac{4}{7}$$

$$\boxed{\theta_{fall} = \arccos\left(\frac{4}{7}\right)}$$

b) Slip: $a_0 < b\alpha = \frac{2}{m}F_f$

$2\frac{F_f}{m} > g\sin\theta - \frac{F_f}{m}$

$3F_f > mg\sin\theta \leftarrow F_f = \mu F_N = F_N = mg\cos\theta - \frac{mv^2}{(R+b)} = mg\left(\frac{7}{3}\cos\theta - \frac{4}{3}\right)$

$7\cos\theta + 4 > \sin\theta$

$7\cos\theta - \sin\theta > 4$ ← solving this relation gives you a θ_{slip}

$$\boxed{7\cos\theta_{slip} - \sin\theta_{slip} > 4}$$