2. A Fresnel rhomb is an optical device used to convert linearly polarized light into circularly polarized light. As shown in Fig. 2, light hits the surface of the rhomb at normal incidence, it then undergoes two total internal reflections inside the rhomb, and then leaves the rhomb again normally.

![Fresnel rhomb](image)

Figure 2: Fresnel rhomb.

The total internal reflections are such that each reflection generates a phase difference of 45° between the component of the light-wave that is parallel and the component that is perpendicular to the plane of incidence (the plane of the page in Fig. 2), and so after two internal reflections a lightwave that was originally linearly polarized at 45° with respect to the plane of incidence becomes circularly polarized.

(a) For a single internal reflection, find the phase shift that the reflected wave acquires relative to the incident wave assuming the electromagnetic wave is polarized in the plane of incidence.

*Hint: Let $r$ be the ratio between the complex amplitude of the reflected wave and that of the incident wave. At total internal reflection, one has $|r| = 1$, so $r = e^{i\phi}$ for some $\phi$.]*

(b) Calculate the phase shift that the reflected wave acquires relative to the incident wave when the electromagnetic wave is polarized perpendicular to the plane of incidence.

(c) If each of the two total internal reflections in a Fresnel rhomb occurs at an angle of incidence of $\theta_i = 53.3°$, calculate the index of refraction $n$ of the Fresnel rhomb relative to that of the surrounding medium.

*You might find useful the following trig. identity: $\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \pm \tan \alpha \tan \beta}$.*

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A) Consider boundary conditions:

1. \( E_i E_i' = E_x E_i' \)
2. \( E_i'' = E_x'' \)
3. \( B_i = B_x \)
4. \( \frac{1}{\mu_i} B_i'' = \frac{1}{\mu_2} B_x'' \Rightarrow B_i'' = B_x'' \)

BC(2) \( \Rightarrow (E_i + E_r) \cos \theta_i = E_x \cos \theta_e \)

BC(4) \( \Rightarrow \frac{1}{\mu_2} (E_i - E_r) = \frac{1}{\mu_2} E_x \Rightarrow \mu_2 (E_i - E_r) = \mu_2 E_x \)

\[
E_c = (E_i + E_r) \frac{\cos \Theta_i}{\cos \Theta_c} \quad \text{and} \quad E_x = \frac{\mu_1}{\mu_2} (E_i - E_r)
\]

\[
(E_i + E_r) \frac{\cos \Theta_i}{\cos \Theta_c} = \frac{\mu_1}{\mu_2} (E_i - E_r) \leftarrow r = \frac{E_c}{E_i}
\]

\[
(r + 1) n_x \cos \theta_i = n_x \cos \theta_x (1 - r)
\]

\[
v(n_x \cos \theta_i + n_x \cos \theta_x) = n_x \cos \theta_x - n_x \cos \theta_i
\]

\[
v = \frac{n_x \cos \theta_x - n_x \cos \theta_i}{n_x \cos \theta_x + n_x \cos \theta_i} \leftarrow n_x = n, \ n_x = 1
\]

\[
\frac{n_x \cos \theta_x - n_x \cos \theta_i}{n_x \cos \theta_x + n_x \cos \theta_i} = r_x + i r_x' \equiv |r_x| e^{i \phi} \Leftarrow \phi = \text{angle at phase shift} \Rightarrow u^{-1} (r_x r_x')
\]

In order to find \( \phi \), need relation between \( \theta_x \) and \( \theta_e \) \( \Rightarrow \) Snell's law

\[
n_i \sin \theta_i = n_x \sin \theta_x \Rightarrow \sin^2 \theta_x = 1 - \cos^2 \theta_x = \frac{n_i^2}{n_x^2} \sin^2 \theta_i \Leftarrow n_i = n, \ n_x = 1
\]

\[
\cos \theta_x = 1 - n_x^2 \sin^2 \theta_i \Rightarrow \cos \theta_x = \sqrt{1 - n_x^2 \sin^2 \theta_i} \Leftarrow n_x \sin \theta_i > 1
\]

\[
\cos \theta_x = i n \sqrt{\sin^2 \theta_i - n_x^2}
\]
\[ r = \frac{i n^4 \sqrt{\sin^4 \theta_i - n^2} - \cos \theta_i}{i n^4 \sqrt{\sin^4 \theta_i - n^2} + \cos \theta_i} = \frac{-(\cos \theta_i - i n^4 \sqrt{\sin^4 \theta_i - n^2})^2}{(\cos \theta_i + i n^4 \sqrt{\sin^4 \theta_i - n^2})(\cos \theta_i - i n^4 \sqrt{\sin^4 \theta_i - n^2})} = \frac{-(\cos^2 \theta_i - n^4 \sin^2 \theta_i + n^2 - 2 n^4 \cos \theta_i \sqrt{\sin^4 \theta_i - n^2})}{\cos^2 \theta_i + n^4 \sin^2 \theta_i - n^2} \]

\[ \begin{cases} 
  r_x = (\cos^2 \theta_i - n^4 \sin^2 \theta_i + n^2)/[-(\cos^2 \theta_i + n^4 \sin^2 \theta_i - n^2)] \\
  r_z = (-2 n^4 \cos \theta_i \sqrt{\sin^4 \theta_i - n^2})/[-(\cos^2 \theta_i + n^4 \sin^2 \theta_i - n^2)] 
\end{cases} \]

\[ \tan \phi_i = \frac{-2 n^4 \cos \theta_i \sqrt{\sin^4 \theta_i - n^2}}{\cos^2 \theta_i - n^4 \sin^2 \theta_i + n^2} = \frac{2 n^4 \cos \theta_i \sqrt{\sin^4 \theta_i - n^2}}{n^4 \sin^2 \theta_i - n^2 - \cos^4 \theta_i} \]

\[ \phi_i = \arctan \left( \frac{2 n^4 \cos \theta_i \sqrt{\sin^4 \theta_i - n^2}}{n^4 \sin^2 \theta_i - n^2 - \cos^4 \theta_i} \right) \]

b) \[ BC \enspace \square \rightarrow (E_i - E_r) = E_t \]

\[ BC \enspace \square \rightarrow \frac{1}{\nu_e} (E_i + E_r) \cos \theta_i = \frac{1}{\nu_e} E_t \cos \theta_e \rightarrow n_1 (E_i + E_r) \cos \theta_i = n_2 E_t \cos \theta_e \]

\[ E_t = \frac{n_1 \cos \theta_i (E_i + E_r)}{n_2 \cos \theta_e} \rightarrow a = \frac{n_1 \cos \theta_i}{n_2 \cos \theta_e} = n \cos \theta_i \left( \frac{i}{n} \right)(\sin^4 \theta_i - n^2)^{5/2} = i \cos \theta_i (\sin^4 \theta_i - n^2)^{5/2} \]

\[ \equiv a(E_i + E_r) = (E_i - E_r) \]

\[ 1 - a = a + ar \rightarrow r(1 + a) = (1 - a) \]

\[ r = \frac{1 - a}{1 + a} = \frac{(1 - a)(1 + a^2)}{(1 + a)(1 + a^2)} \rightarrow a^2 = -a \]

\[ a = \frac{(1 - 2a + a^2)}{(1 - a^2)} \]

\[ \tan \phi_i = \frac{-2 a}{1 + a^2} \]
\[\phi_i = \tan^{-1}\left(\frac{2 \cos\theta_i (\sin^2\theta_i - n^2)^{-\frac{1}{2}}}{1 - \cos^2\theta_i (\sin^2\theta_i - n^2)^{-1}}\right)\]

c) \[\phi_\parallel - \phi_\perp = \frac{\pi}{4}\]

\[\tan(\phi_\parallel - \phi_\perp) = 1 = \frac{\tan \phi_\parallel - \tan \phi_\perp}{1 + \tan \phi_\parallel \tan \phi_\perp}\]

Solving this condition at \(\theta_i = 53.3^\circ\) will give you \(n\).

I don't have the time to do this.

Sorry :(