Section B. Electricity and Magnetism

1. A small wire loop of radius \( a \) lies in the \( xy \)-plane, centered on the origin. A magnetic moment \( \mathbf{m} = m \hat{z} \) travels up along the \( z \) axis with constant speed \( v \). It passes through the center of the wire loop at \( t = 0 \).

(a) Compute the emf \( \mathcal{E}(t) \) around the loop.

   \textit{Hint: the integral is easier if you evaluate the flux through a section of a spherical surface centered on the magnet and bounded by the wire loop rather than through the planar area bounded by the loop.}

(b) If the loop has resistance \( R \), find the Joule heat \( P(t) \) Assume the loop is fixed in position.

(c) Now consider the case where a uniform linear charge density \( \lambda \) is glued to a non-conducting loop (same orientation and radius as above), and the loop is allowed to spin. What is the position of \( \mathbf{m} \) at the time the loop attains its largest angular momentum, \( L_{\text{max}} \)? What is the value of \( L_{\text{max}} \)? Assume the dipole began its constant-velocity motion at \( t = -\infty \), and that the loop was at rest then.
\[ \theta_c = \arcsin \left( \frac{2v}{\sqrt{a^2 + 2v^2}} \right) = \arcsin \left( \frac{v}{\sqrt{2}} \right) \]

\[ \varepsilon = -\frac{F - \bar{F}}{2t} \leq \Phi = \int \int \int \nabla \cdot \mathbf{B} \, dV \]

\[ \mathbf{a} = r^2 \sin \theta \, d\theta \, d\phi \]

\[ \mathbf{n} = \mathbf{r} = \mathbf{v}_n \]

\[ \mathbf{r} = \mathbf{v}_n \tan \theta \, d\theta \, d\phi \]

\[ \Phi = \frac{\text{Mom}}{2 \pi r} \int_{0}^{2\pi} \int_{0}^{\pi/2} \sin \theta \cos \theta \sin \theta \, d\theta \, d\phi \]

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\[ x = \sin \theta \quad d\phi = \cos \theta \, d\phi \quad \int_{0}^{\pi/2} \sin \theta \, d\theta = 1 \]

\[ \frac{\text{Mom} a^2}{2} = \frac{\text{Mom} a^2}{2r^2} \left( -\frac{3}{2} \right) (a^2 + 2v^2)^{3/2} = \frac{3 \text{Mom} a^2 v^2 t}{2(a^2 + 2v^2)^{3/2}} \]

\[ \varepsilon = \frac{3 \text{Mom} a^2 v^2}{2(a^2 + 2v^2)^{3/2}} \]
6) $P = I \varepsilon \Rightarrow I = \frac{\varepsilon}{R} = \frac{e^2}{R} = \frac{3\mu_0 m a^2 v^2}{2(\alpha^2 + v^2)^{3/2}} \Rightarrow \lambda = \frac{g m a^2 v^2}{4(\alpha^2 + v^2)^{3/2}}$

$P(t) = \frac{g m a^2 v^2}{4(\alpha^2 + v^2)^{3/2}}$

C) $\varepsilon = \int_{-\infty}^{\infty} E(t) dt = 2\pi m E \rightarrow |E| = \frac{\varepsilon}{2\pi a}$

$\frac{2\varepsilon}{\pi a} = \frac{2\varepsilon}{\pi a} = \frac{4}{\pi} \int_{-\infty}^{\infty} E(t) dt = \frac{\varepsilon}{\pi a} \Rightarrow E(t) = \frac{3\lambda m a^2 v^2}{2(\alpha^2 + v^2)^{3/2}}$

$\max \text{ when } \frac{dE}{dt} = 0 \Rightarrow t_{max} = 0$

$t_{max} = 0$

$\frac{dE}{dt} = \lambda a \varepsilon \frac{\alpha}{2} = -\lambda a \frac{2 \pi a}{2}$

$I = -\lambda a \frac{\alpha}{2} \frac{\varepsilon}{2} = -\lambda \frac{2 \pi m a^2}{2} (\alpha^2 + v^2)^{-3/2} \Rightarrow$

$I(t_{max}) = -\lambda \frac{2 \pi m a^2}{2} (\alpha^2)^{1/2} = -\lambda \frac{2 \pi m a}{2}

\tilde{I}_{max} = -\frac{2 \pi m a}{2}$