1. A particle of mass \( m \), with the Hamiltonian \( H = \frac{p^2}{2m} + V(x) \), is moving in one dimension subject to an attractive potential of the form:

\[
V(x) = -U \left[ \delta(x + a/2) + \delta(x - a/2) \right]
\]

with \( U > 0 \).

(a) What consequences does the Hamiltonian’s reflection symmetry have for the particle’s bound states?

(b) For \( U \) large enough the Hamiltonian has two bound states. Sketch their wave functions, making it clear which describes the ground state.

(c) For \( U \leq U_c \) the Hamiltonian has only one bound state. Determine the value of \( U_c \), in terms of the other parameters.
a) It implies that the bond states are either symmetric or antisymmetric.

b) The wave function is given by:

\[ \psi(x) = \begin{cases} 
A e^{kx} & x < -l \\
B e^{kx} + C e^{-kx} & -l < x < l \\
D e^{-kx} & x > l 
\end{cases} \]

where \( k =\sqrt{-2mE}/\hbar \) and \( E < 0 \) for bound states.

Continuity at \( x = \pm l \):

\[ A e^{-kx} = B e^{-kx} + C e^{kx} \rightarrow A = B + C e^{2kx} \]

\[ B e^{kx} = B e^{kx} + C e^{-kx} \rightarrow D = B e^{2kx} + C \]

\[ \psi(x) = \begin{cases} 
B e^{kx} + C e^{kx} & x < -l \\
B e^{kx} + C e^{-kx} & -l < x < l \\
B e^{-kx} + C e^{-kx} & x > l 
\end{cases} \]

Once obtaining the W.F., an energy relation can be found from boundary conditions at the delta functions.
\[-\frac{\mu_k}{2\mu^3} \gamma''(x) - 2\mu_u \gamma(x) - \pi \delta(x-L) \gamma(x) = E \gamma(x)\]

\[-\frac{\mu_k}{2\mu^3} \int_{-L}^{L} \gamma''(x) - 2\mu_u \gamma(x) - \pi \delta(x-L) \gamma(x) \, dx = \int_{-L}^{L} E \gamma(x) \, dx\]

\[-\frac{k}{2\mu} [\gamma'(-L) - \gamma'(L)] - u_0 \gamma(L) = 0\]

\[\gamma'(L) = -\frac{k}{2\mu} [\gamma'(-L) - \gamma'(L)]\]

\[\gamma'(L) - \gamma'(-L) = k [(-B_0 e^{kx} - c^{-ke}) - (B_0 e^{-ke} - c^{ke})] = -2kB_0 e^{kx}\]

\[\gamma'(-L) - \gamma'(-L) = k [(B_0 e^{-ke} - c^{ke}) - (B_0 e^{ke} + c^{ke})] = -2kC e^{kx}\]

at \(x = -L\):

\[B e^{-ke} + C e^{ke} = -\frac{k}{2\mu} (-2kC e^{kx}) = \frac{k}{\mu} C e^{kx}\]

\[B = C (\frac{k}{\mu} - 1) e^{kx}\]

at \(x = L\):

\[B e^{ke} + C e^{-ke} = -\frac{k}{2\mu} (-2kB_0 e^{kx}) = \frac{k}{\mu} B e^{kx}\]

\[C = B (\frac{k}{\mu} - 1) e^{kx} = C (\frac{k}{\mu} - 1)^x e^{kx}\]

\[(\frac{k}{\mu} - 1)^x = e^{-4ke} \Rightarrow \frac{k}{\mu} - 1 = e^{-2ke}\]

\[\frac{n+1}{\mu u} \kappa = 1 \pm e^{-2ke}\]
Need to find when \( \frac{\hbar^2}{mn^2} K = 1 - e^{-x^2} \) for \( k < 0 \)

take for high-energy state where \( E \approx 0 \rightarrow k \ll 1 \)

\[
\frac{\hbar^2}{mn^2} K = 1 - e^{-x^2} = 1 - (1 - 2Kl) = 2Kl = Ka \Rightarrow \]

\[
U_c = \frac{\hbar^2}{mn^2} \]

\[
U_c = \frac{\hbar^2}{mn^2} \]