

## J14M.3 Solution

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This is classic small oscillations. First, let's write a Lagrangian.

$$T = \frac{1}{2}I_{\theta}\dot{\theta}^2 + \frac{1}{2}I_{sphere}\dot{\phi}^2$$
$$V = -mg(R-b)\cos\theta$$

the non-slip condition gives  $\dot{\phi}b = \dot{\theta}(R-b)$ . A lot of people have questioned this non-slip condition but this is definitely correct. To see why, think about the change in the contact point if the ball moved an angle  $\theta$  along the cylinder, while entirely slipping. The contact point would move a distance  $b\theta$ . Thus when we add in rolling, the distance on the ball is  $b\theta + b\phi$  while the distance on the cylinder is just  $R\theta$ . Equating these two and taking the derivative gives the stated condition

$$\mathcal{L} = \frac{1}{2}m(R-b)^2\dot{\theta}^2 + \frac{1}{5}mb^2\dot{\theta}^2\left(\frac{R-b}{b}\right)^2 + mg(R-b)\cos\theta$$

From this we get the equation of motion in  $\theta$

$$\left(m(R-b)^2 + \frac{2}{5}mb^2\left(\frac{R-b}{b}\right)^2\right)\ddot{\theta} = -mg(R-b)\sin\theta$$

Small angle approximation gives us  $\sin\theta \approx \theta$

$$\ddot{\theta} + \frac{mg(R-b)}{m(R-b)^2 + \frac{2}{5}mb^2\left(\frac{R-b}{b}\right)^2}\theta = 0$$
$$\ddot{\theta} + \frac{g}{(R-b) + \frac{2}{5}(R-b)}\theta = 0$$
$$\Rightarrow \omega = \sqrt{\frac{5g}{7(R-b)}}$$