2. This is the dawn of the age of Aquarius, due to the precession of the Earth’s spin axis $\Omega$ around the celestial orbital axis $\hat{z}$. The Earth is slightly elliptical due to its spin. Approximate the Earth as a perfect sphere of radius $R_0$ and mass $M$, but assume a thin ring of radius $R_e$ with mass $\delta M$ is in the place of the equator of the Earth. The Sun has mass $M_s$ and is a distance $R_{se}$ from the center of the Earth to the Sun. Assume that $R_e \ll R_0$, $R_{se} \approx R_0$, and $\delta M \ll M$.

(a) From the figure, what is the torque $\tau$ acting on the Earth about its center of mass due to the Sun? You’ll need to do some approximations to get a tractable answer. One approximation is to use only the $y$ coordinate in estimating how far each point on the ring is from the Sun.

(b) Neglecting the effects of the Moon’s gravity, what is the rate of precession $\omega_p$ of the angular momentum $L$ of the Earth around the celestial axis? If you couldn’t solve part (a), then assume the torque is $\tau$, where you don’t need to know the magnitude of the vector $|\tau| = \tau_p$, but you should know the direction. You may take the magnitude of the angular momentum, $L$, as known.
b) \[ \dot{\mathbf{r}} = \frac{dL}{dt} \]

\[ C_x = \left( \frac{dL}{db} \right)_x = \frac{dL}{db} \frac{do}{dt} = L \sin \theta \frac{do}{d\theta} = L \omega_p \sin \theta \]

\[ L = I \Omega = \frac{2}{5} MR^2 \Omega \]

\[ \omega_p = \frac{C_x}{\frac{2}{5} MR^2 \Omega \sin \theta} \]

\[ \Omega_p = \frac{5}{2} \frac{GSM}{R_e} \left( \frac{R_0^2}{R_e^2} \right) \cos \theta \]

\[ \frac{R_0}{R_e} \Omega = \frac{5}{2} \frac{GSM}{R_e^3} \Omega \cos \theta \]