J14E.2 Solution

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a) Finding the flux is a little trickier than it first appears to be since the magnetic field has a discontinuity at the center. Thus it's easier to work with the vector potential.

$$\Phi = \int \vec{B} \cdot d\vec{A}$$
$$\Phi = \int \nabla \times \vec{A} \cdot d\vec{A}$$
$$\Phi = \int \vec{A} \cdot d\vec{l}$$

Now we remember our formula for the vector potential of a magnetic dipole. I think it's best to memorize this and the formula for the magnetic field of the magnetic dipole for prelims, as it comes up a lot and is somewhat time consuming to derive on the spot.

$$\vec{A} = \frac{\mu_0 \vec{m} \times \hat{r}}{4\pi r^2}$$
$$\Rightarrow \Phi = \frac{\mu_0}{4\pi R^2} \int m\hat{z} \times \hat{r} \cdot R d\phi \hat{\phi}$$
$$\Phi = \frac{\mu_0 m \sin\theta}{2R}$$

Since the dipole is in the plane of the loop, we evaluate the flux at $\theta = \pi/2$

$$\Phi = \frac{\mu_0 m}{2R}$$

b) This requires finding the energy stored in the magnetic dipole in a magnetic field, given by

$$U = -\vec{m} \cdot \vec{B}$$

where the B-field here is the magnetic field from the current loop. Using Biot-Savart law, I got that

$$\vec{B}_{loop} = \frac{\mu_0}{4\pi}\int \frac{IRd\phi\hat{\phi} \times -\hat{r}}{R^2}$$
$$\vec{B}_{loop} = \frac{\mu_0 I}{2R}\hat{z}$$

Thus the initial energy in the magnetic moment is

$$U_i = -\frac{\mu_0 Im}{2R}$$

While turning the magnetic moment 180 degrees would give an energy of

$$U_f = \frac{\mu_0 Im}{2R}$$

Thus the energy that must be exerted to turn it is

$$W = U_f - U_i = \frac{\mu_0 Im}{R}$$

c) We need to find the difference in the power during the time the magnetic moment was flipped. Once we find that we can integrate across all time to get the total energy supplied. To figure out the power, we first need the change in the voltage due to the flipping. Using Faraday's law we get

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$
$$\int \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t}\int \vec{B} \cdot d\vec{A}$$
$$V = -\frac{\partial}{\partial t}\int \vec{A} \cdot d\vec{l}$$
$$P = IV = -I\frac{\partial}{\partial t}\int \vec{A} \cdot d\vec{l}$$

Then integrating from before the flip started to afterwards, we get

$$U_E = \left|\int Pdt\right| = \int I\frac{\partial}{\partial t}\int \vec{A} \cdot d\vec{l}dt$$
$$U_E = I\Delta\left(\int \vec{A} \cdot d\vec{l}\right)$$

Using the answer from part a), we know what the integral will be before and after the flip

$$U_E = \frac{\mu_0 mI}{R}$$

Thus we see that the extra energy supplied by the circuit is the energy that it takes to flip the magnetic moment.