Problem J14E.2

(a) We use a mutual inductance argument.

Approximate the particle as a small circular loop with radius r and current $I = m/(\pi r^2)$. We will take $r \to 0^+$ at the end.

Suppose there is a current I' flowing through the large loop. The B-field at the particle's location is

$$B = \frac{\mu_0 I'}{2R},$$

so the flux through the small loop is

$$\Phi_B = \pi r^2 B = \mu_0 \pi r^2 \frac{I'}{2R}.$$

Hence the mutual inductance of the two loops is

$$M = \frac{\Phi_B}{I'} = \mu_0 \frac{\pi r^2}{2R}.$$

By symmetry of mutual inductance, the flux through the large loop due to the small loop is

$$\Phi'_B = MI = \boxed{\frac{\mu_0 m}{2R}}$$

as desired.

(b) The B-field at the center of the loop is $B = \mu_0 I/2R$. Thus the energy required to flip the dipole is

$$2mB = \frac{\mu_0 m I}{R}.$$

(c) The EMF induced by the rotating dipole is equal to

$$\mathcal{E} = -\frac{\partial \Phi_B}{\partial t},$$

which consumes an electrical energy

$$E = \int I\mathcal{E}\,\mathrm{dt} = -I\left(\Phi_{B,f} - \Phi_{B,i}\right).$$

From part (a), this expression simplifies to

$$E = -\frac{\mu_0 m I}{R},$$

which agrees with the energy from part (b).