Problem J14E.1

Inside and outside the cylinder, we must be able to decompose the magnetic scalar potential Ψ as a linear combination of cylindrical harmonics of the form

$$c_n = \cos(n\phi)\rho^n$$

where ϕ is the azimuthal angle. The corresponding terms with $\sin(n\phi)$ must vanish by reflection symmetry.

(There is also the possibility of a $\log\rho$ dependence, but it turns out not to be necessary.)

Inside the cylinder, we must have

$$\Psi = \alpha m_0 x = \alpha m_0 c_1$$

for some coefficient α. Matching boundary conditions on the cylinder forces

$$\Psi = \alpha m_0 R^2 c_{-1} = \alpha m_0 R^2 \rho^{-1}\cos\phi$$

outside the cylinder, since we must have $\Psi \to 0$ as $\rho \to \infty$. Thus we compute

$$\mathbf{H} = -\boldsymbol{\nabla}\Psi = -\alpha m_0 \hat{\mathbf{x}}$$

inside the cylinder, and

$$\mathbf{H} = -\boldsymbol{\nabla}\Psi = \alpha m_0 R^2 \rho^{-2}\left(\cos\phi\,\hat{\boldsymbol{\rho}} + \sin\phi\,\hat{\boldsymbol{\phi}}\right)$$

outside the cylinder. Inside the cylinder, it follows that

$$\mathbf{B} = \mu_0\left(\mathbf{H} + \mathbf{M}\right) = \mu_0 m_0 \left(1-\alpha\right)\hat{\mathbf{x}},$$

and outside the cylinder we have $B = \mu_0\mathbf{H}$. The B-field must be continuous, and enforcing this condition at $\phi = 0$ yields

$$\alpha\,\hat{\boldsymbol{\rho}} = \left(1-\alpha\right)\hat{\mathbf{x}},$$

so $\alpha = 1/2$.