J13E.2 (Radiat^g Fields)

(a) Assume $\bar{c}$ radius of $\bar{c}$ atom is $\bar{c}$ classical orbit radius.

Balanc^g $\bar{c}$ electric & centripetal forces: $\frac{e^2}{4\pi\varepsilon_0 D^2/4} = m_e \frac{D}{2}\omega_0^2$

$$\Longrightarrow \omega_0^2 = \frac{2e^2}{\pi\varepsilon_0 D^3 m_e} \quad \Longrightarrow \omega_0 = \sqrt{\frac{2e^2}{\pi\varepsilon_0 D^3 m_e}}$$

(b) This will turn into an oscillat^g electric dipole w/ dipole moment:

$\vec{p} = \frac{1}{2} eD\cos(\omega_0 t)\,\hat{z}$, where we can WLOG set $\bar{c}$ dipole oscillat^g along $\hat{z}$.

$$\vec{A}^{ED}(\vec{r}, t) = \frac{\mu_0}{4\pi r}\dot{\vec{p}}(t - \tfrac{r}{c})$$

$$\dot{\vec{p}} = -\frac{1}{2} eD\omega_0 \sin(\omega_0 t)\,\hat{z}$$

$$\Longrightarrow \vec{A}^{ED}(\vec{r}, t) = \frac{\mu_0}{4\pi r}\cdot\frac{1}{2} eD\omega_0 \sin\left(\omega_0 t - \frac{\omega_0 r}{c}\right)(\cos\theta\,\hat{r} - \sin\theta\,\hat{\theta})$$

$$\vec{B} = \nabla\times\vec{A}$$

$$= \frac{1}{r}\left(\frac{\partial(rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial\theta}\right)\hat{\phi}$$

$$= \frac{1}{r}\left\{\frac{\mu_0}{8\pi} eD\omega_0 \sin\theta\,\frac{\omega_0}{c}\cos\left(\omega_0 t - \frac{\omega_0 r}{c}\right) + \frac{\mu_0}{8\pi r} eD\omega_0\sin\theta\sin\left(\omega_0 t - \frac{\omega_0 r}{c}\right)\right\}\hat{\phi}$$

$\because$ we only want $\bar{c}$ far field behaviour, r is large ($r \gg \lambda \Longrightarrow \frac{1}{r} \ll k = \frac{\omega_0}{c}$), so drop $\bar{c}$ 2^{nd} term.

$\Longrightarrow \vec{B} = \frac{\mu_0}{8\pi r c} eD\omega_0^2 \sin\theta\cos\left(\omega_0 t - \frac{\omega_0 r}{c}\right)\hat{\phi} \equiv \frac{\mu_0}{4\pi r c} p_0\,\omega_0^2 \sin\theta\cos[\omega_0(t - \frac{r}{c})]\hat{\phi}$, which looks like Griffith's!

$\vec{E} = c\hat{r}\times\vec{B} = \frac{-\mu_0}{4\pi r} p_0\,\omega_0^2\sin\theta\cos[\omega_0(t-\frac{r}{c})]\hat{\theta}$ ($\bar{c}$ signs don't matter for $\bar{c}$ power).

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$$

$$= \frac{\mu_0}{16\pi^2 r^2 c} p_0^2\,\omega_0^4 \sin^2\theta\cos^2[\omega_0(t - \tfrac{r}{c})]\,\hat{r}$$

$$\langle\vec{S}\rangle_t = \frac{\mu_0 p_0^2\omega_0^4}{32\pi^2 r^2 c}\sin^2\theta\,\hat{r}$$

$$\Longrightarrow P_{rad} = \int d\vec{\Omega}\cdot\langle\vec{S}\rangle$$

$$= \frac{\mu_0 p_0^2\omega_0^4}{32\pi^2 c}\int d\theta\,\sin^3\theta\cdot 2\pi$$

$$= \frac{\mu_0 p_0^2\omega_0^4}{16\pi c}\int d\theta\,\sin^3\theta$$

$$= \frac{\mu_0 p_0^2\omega_0^4}{12\pi c}$$

I'm guess^g $\bar{c}$ average power is just $I_{tot} = \frac{P_{rad}}{4\pi} = \frac{\mu_0 p_0^2\omega_0^4}{48\pi c}$ (average power per unit solid angle).

(c) Suppose ē radiated power is const. over time (at a given radius).

Then, as a funct^{n} of radius: $\dfrac{e^2}{4\pi\varepsilon_0 r^3 m_e} = \omega^2(r)$

$$\frac{dE}{dt} = \frac{\mu_0 p_0^2}{12\pi c} \cdot \omega^4 = \frac{dE}{dr} \cdot \frac{dr}{dt} = m_e r \omega^2 \cdot \frac{dr}{dt}$$

$$\frac{\mu_0 p_0 e^2}{48\pi^2 \varepsilon_0 c m_e^2} \, dt = r^4 \, dr$$

$$\frac{\mu_0 p_0 e^2}{48\pi^2 \varepsilon_0 c m_e^2} \tau = \frac{1}{5}\left(\frac{D}{2}\right)^5 \Longrightarrow \tau = (\ldots)$$, where $(\ldots)$ is someth^{g} I don't even know is right.