

Prelims Solutions

Problem J12M2

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Using torques gets tough quickly. Use energy conservation. The initial energy is essentially 0, the kinetic energy comes from the falling part of the towel and the rotation of the (roll + remaining towel), the potential energy is of the fallen towel. Thus,

$$0 = \frac{1}{2}(I_{roll} + I_{remainingtowel})\dot{\theta}^2 + \frac{1}{2}(\lambda\theta R)(\dot{\theta}R)^2 - (\lambda\theta R)g\frac{\theta R}{2}$$

where $I_{roll} = M_{roll}R^2$ and $I_{remainingtowel} = (m - \lambda\theta R)R^2$. Hence,

$$\dot{\theta} = \sqrt{\frac{\lambda g \theta^2}{M_{roll} + m - \frac{1}{2}\lambda\theta R}}$$