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Consider a two-level system with two orthogonal states $|g\rangle$ and $|e\rangle$. It has time-dependent Hamiltonian:

$$H(t) = \hbar\omega\,|e\rangle\langle e| + V cos(\omega t)(|e\rangle\langle g| + |g\rangle\langle e|)$$

Assume that the time-dependent term is small, $\hbar\omega >> V > 0$, so that you may make the corresponding approximation. At time $t = 0$ the state of the system is specified by the initial complex amplitudes c_{g0} and c_{e0}:

$$|\Psi(t = 0)\rangle = c_{g_0}\,|g\rangle + c_{e_0}\,|e\rangle$$

What is the state of the system $|\Psi(t)\rangle$ at all other times t?

(We set $\hbar = 1$) Since the time-dependent term is small we can treat the problem in time-dependent perturbation theory. We call the time-independent part of the Hamiltonian, $H_0 = \omega\,|e\rangle\langle e|$ and the time-dependent part of the Hamiltonian $V(t) = V cos(\omega t)(|e\rangle\langle g| + |g\rangle\langle e|)$. Now we want to find the following state:

$$|\Psi(t)\rangle = c_g(t)\,|g\rangle + c_e(t)\,|e\rangle$$

$c_g(t)$ and $c_e(t)$will be the transition amplitudes from the initial state to $|g\rangle$ and $|e\rangle$, respectively, in time-dependent perturbation theory.

$$c_g(t) = \langle g|\,(1 - i\int_0^t V_I(t)dt)\,|\Psi_0\rangle$$

$$c_g(t) = c_{g_0} - i\int_0^t \langle g|\,V(t)(c_{g_0}\,|g\rangle + c_{e_0}e^{-i\omega t}\,|e\rangle)$$

Here we have used the fact that in the interaction picture our time-dependent potential looks like:

$$V_I(T) = e^{iH_0 t}V(t)e^{-iH_0 t}$$

Thus we now have that:

$$c_g(t) = c_{g_0} - i\frac{Vc_{e_0}}{2}\int_0^t (e^{i\omega t} + e^{-i\omega t})e^{i\omega t}dt$$

$$c_g(t) = c_{g_0} - i\frac{Vc_{e_0}}{2}(t - \frac{1}{2i\omega}e^{-2i\omega t})$$

Similarly we can find that:

$$c_e(t) = c_{e_0} - i\frac{Vc_{g_0}}{2}(t + \frac{1}{2i\omega}e^{2i\omega t})$$