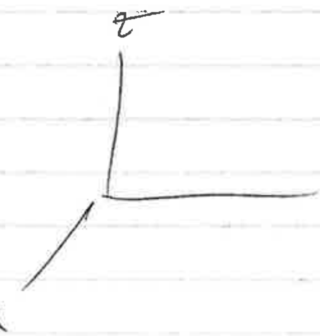
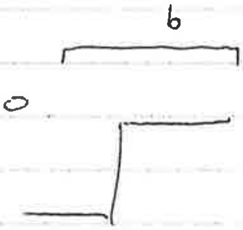
j12 e.1



$V(y,z)$

$V(y,0) = 0$

$V_0(y) = V(y,a) =$



wave

$\nabla^2 V = 0 \qquad V = \cancel{X(x)}\, Y(y)\, Z(z)$

$\frac{Z''}{Z} + \frac{Y''}{Y} = 0$

odd, with period b.

periodic in y, so $\frac{Y''}{Y} = -c^2 \qquad Y = \cancel{A\cos ay} + B\sin ay$

$Y = \sum A_n \sin\left(\frac{2\pi n y}{b}\right)$

$\frac{Z''}{Z} = c^2 \rightarrow Z = Be^{cz} + Ce^{-cz}$

$Z(0) = 0 = B + C$

$Z = B\left(e^{cz} - e^{-cz}\right) = B\sinh(cz)$

$$V(y,z) = \sum_{n=1}^{\infty} A_n \sinh\left(\frac{2\pi n z}{b}\right) \sin\left(\frac{2\pi n y}{b}\right)$$

$$V(y,a) = \sum_{n=1}^{\infty} A_n \sinh\left(\frac{2\pi a}{b} n\right) \sin\left(\frac{2\pi n}{b} y\right)$$

$$\int_0^b \sin\left(\frac{2\pi m}{b} y\right) V_0(y)\, dy = A_m \sinh\left(\frac{2\pi a m}{b}\right) \int_0^b \sin^2\left(\frac{2\pi m}{b} y\right) dy$$

$$V_0 \int_0^{\frac{b}{2}} \sin\left(\frac{2\pi m}{b} y\right) dy - V_0 \int_{\frac{b}{2}}^{b} \sin\left(\frac{2\pi m}{b} y\right) dy = A_m \sinh\left(\frac{2\pi a m}{b}\right) \frac{b}{2}$$

$$-\frac{b V_0}{2\pi m} \cos\left(\frac{2\pi m}{b} y\right)\Big|_0^{\frac{b}{2}} + V_0 \frac{b}{2\pi m} \cos\left(\frac{2\pi m}{b} y\right)\Big|_{\frac{b}{2}}^{b}$$

$$-\frac{b V_0}{2\pi m}\left((-1)^m - 1\right) + \frac{V_0 b}{2\pi m}\left(1 - (-1)^m\right)$$

$$\frac{V_0 b}{\pi} \frac{\left((-1)^m - 1\right)}{m} = A_m \sinh\left(\frac{2\pi a}{b} m\right) \frac{b}{2}$$

$$-\frac{2V_0}{\pi} \frac{1}{m \sinh\left(\frac{2\pi a}{b} m\right)} = A_m \quad m \text{ odd}, \quad A_m = 0 \quad m \text{ even}$$