Problem J12E.3

(a) We may assume that the particle sits at location $(0, 0, z)$ with magnetic moment

$$\mathbf{m} = (m \sin\theta, 0, m\cos\theta).$$

Then we may replace the superconductor with an image dipole $\mathbf{m}' = (m\sin\theta, 0, -m\cos\theta)$ at $(0, 0, -z)$, so that the $\mathbf{B}$-field is purely tangential at the surface $z = 0$.

The interaction energy of the dipole and its image is

$$\begin{aligned} E_{\text{int}} &= -\frac{1}{2}\frac{\mu_0}{4\pi(2z)^3}(3m_z m_z' - \mathbf{m}\cdot\mathbf{m}') \\ &= \frac{1}{2}\frac{\mu_0 m^2}{4\pi(2z)^3}(3\cos^2\theta - \cos 2\theta), \end{aligned}$$

where the factor of $1/2$ accounts for the fact that the image dipole is induced (i.e. the energy of the B-field for $z < 0$ is fictitious).

Using the identity

$$\begin{aligned} 3\cos^2\theta - \cos 2\theta &= \frac{3}{2}(1 + \cos 2\theta) - \cos 2\theta \\ &= \frac{3}{2} + \frac{1}{2}\cos 2\theta, \end{aligned}$$

we find that the total energy $U = E_{\text{int}} + Mgz$ is minimized when $\theta = \pi/2$ and

$$0 = \frac{\partial E}{\partial z} = -3\frac{\mu_0 m^2}{64\pi} z^{-4} + Mg,$$

which rearranges to

$$z = \boxed{\sqrt[4]{\frac{3\mu_0 m^2}{64\pi M g}}}.$$

As a sanity check, a $1\,\text{cm}^3$ neodymium magnet has a magnetic moment of about $1\,\text{J/T}$. Using a weight of $10\,\text{g}$, we obtain a floating height of $2\,\text{cm}$, which is about right.

(b) As derived in part (a), the magnetic moment $\mathbf{m}$ is perpendicular to the z-axis.