Prelims Solutions

Problem J11T3

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1

On the coexistence curve the chemical potential of the liquid and gas are equal $\mu_g = \mu_l$. So $d\mu_g = d\mu_l \rightarrow \frac{\partial \mu_g}{\partial P} dP + \frac{\partial \mu_g}{\partial T} dT = \frac{\partial \mu_l}{\partial P} dP + \frac{\partial \mu_l}{\partial T} dT$. Hence,

$$\frac{dP}{dT} = \frac{\frac{\partial \mu_g}{\partial T} - \frac{\partial \mu_l}{\partial T}}{\frac{\partial \mu_l}{\partial P} - \frac{\partial \mu_g}{\partial P}} = \frac{s_g - s_l}{v_l - v_g}$$

using Maxwell relations for the Gibbs potential $G = \mu N$, where $s = \frac{S}{N}$ and $v = \frac{V}{N}$. The latent specific heat is defined as $l = \frac{L}{Nm} = \frac{\Delta Q}{Nm} = \frac{T \Delta S}{N} = T(s_g - s_l)/m$ where m is the mass of the molecule. Now:

$$\frac{dP}{dT} = \frac{l}{T(v_l - v_g)/m} = \frac{l}{T(\frac{1}{\rho_l} - \frac{1}{\rho_g})}$$

2

Using

$$\frac{\Delta P}{\Delta T = -5K} = \frac{3 * 10^5 J/kg}{273K * (-10^{-4} m^3/kg)} \rightarrow \Delta P = 5 * 10^7 Pa$$

3

Pressure on the skate of said ice skater is: $\frac{70kg * 10m/s^2}{.3m * .005m} \approx 5 * 10^5 Pa$. This is a factor of 100 too small, yet little kids can easily skate on ice. Freezing point depression of normal ice obviously doesn't support this theory of melted ice causing the low friction. To keep this theory afloat, maybe the top layers of an ice surface don't have the same structure (not as ordered) as the bulk and so might have a lower l.