Prelims Solutions

Problem J11T1

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1

Adiabatic conditions mean $dQ = 0$ so the first law gives $dU = -PdV$ where $U = C_v k_b T = f/2Nk_bT$ for an ideal gas. Using $PdV + VdP = Nk_bdT$ we have

$$dU = f/2Nk_bdT = -PdV = VdP - Nk_bdT$$

$$\rightarrow (f/2+1)Nk_bdT = \frac{Nk_bT}{P}dP$$

$$\rightarrow T^\beta = constant * P$$

where $\beta = \frac{f+2}{2}$.

2

Consider the pressure change over a thin slab of air of height dz and area A. The extra pressure has to balance the extra weight of the air in the slab. Hence, $dPA = -\rho Adzg \rightarrow \frac{dP}{dz} = -\rho g$ with z positive above the ground.

3

$\frac{dT}{dz} = \frac{dT}{dP}\frac{dP}{dz}$. We use $\rho = \frac{MN}{N_A V} = \frac{MP}{N_A k_b T} \rightarrow \frac{dP}{dz} = -\frac{MP}{k_b T}g$ and $T = cP^{1/\beta} \rightarrow \frac{dT}{dP} = \frac{1}{\beta}c\frac{P^{1/\beta}}{P}$ to get

$$\frac{dT}{dz} = (\frac{1}{\beta}c\frac{P^{1/\beta}}{P})(-\frac{MP}{N_A k_b T}g) = -\frac{Mg}{\beta R}.$$

4

We know $T(z) = -\frac{Mg}{\beta R}z + T_o$ so by scaling our adiabatic relation between P and T from part a:

$$P = (\frac{T(z)}{T_o})^\beta * P_o$$