J11M1 - Lasso (Solution by Jim Wu)

A lasso is a rope of linear mass density ρ that ends in a loop called the honda. The free end of the rope is fed through the honda to create a large loop called the noose. The remaining length of rope is called the spoke, which is used to impart energy to the noose (spin it up) and to support it against the downward pull of gravity. Consider the case of a circular noose of radius r spinning in a nearly horizontal plane with angular speed ω (you can neglect the tilt angle of the noose with respect to the horizontal). The spoke is supported above the center of the noose and makes an angle θ with respect to the vertical. There is no friction between the rope and the honda.

(a) What is the tension in the rope within the loop?

(b) The inward force of the spoke on the honda tends to perturb the shape of the noose near the honda (as can be seen in Dick Corys picture, above). To counteract this effect, the honda is given an additional mass m_h (say, by wrapping it with a heavy metal wire). What should m_h be in terms of the given quantities in order to maintain the circular shape of the noose?

(c) Suppose the spinning noose is subject to a small-amplitude transverse disturbance (a kink) that results in a wave propagating along the rope in the direction opposite to the rotation of the noose. What is the angular speed of the wave as viewed from a reference frame at rest with respect to the spinning noose? What does your answer imply about the motion of the kink as seen by an observer in the fixed frame?

Solution:

(a) Consider a small segment of the rope subtended by an angle $d\phi$, and therefore has a mass $dm = \rho R d\phi$. The tension on the left is $T(\phi)$ and on the right end is $T(\phi + d\phi)$. After a bit of geometry, one finds that the angle between the horizontal and the tension vector is $d\phi/2$. So, the radial force is

$$F_r = T(\phi)\sin\frac{d\phi}{2} + T(\phi + d\phi)\sin\frac{d\phi}{2} = (\rho R d\phi)\omega^2 R$$

From small angle approximation $\sin\frac{d\phi}{2} \approx \frac{d\phi}{2}$ and keeping only up to first order in $d\phi$, we find that

$$2T(\phi)\left(\frac{d\phi}{2}\right) = \rho\omega^2 R^2 d\phi \qquad \Rightarrow \qquad T(\phi) = \rho\omega^2 R^2$$

Note that the tension does not depend on the angle, which makes sense by rotational symmetry of the circular rope.

(b) To keep the lasso in a circular shape, the tension from the spoke must provide the centripetal acceleration to keep the honda moving in the circle. So, from Newton's second law,

$$T\sin\theta = m_h\omega^2 R$$

The tension is the same as that of the circular rope at the point of the honda, and so

$$\rho\omega^2 R^2\sin\theta = m_h\omega^2 R \qquad \Rightarrow \qquad m_h = \rho R\sin\theta$$

(c) The speed of a wave propagating relative to the rope is given by

$$v = \pm\sqrt{\frac{T}{\rho}} = \pm\sqrt{\omega^2 R^2} = \pm\omega R$$

A wave propagating in the direction of the rotation of the noose is has angular speed ω, and a wave going in the opposite direction of rotation has an angular speed of $-\omega$. Since the noose is spinning at ω, then the counter-propagating kink appears stationary with $\omega_{fixed} = 0$ to an observer in the fixed frame.

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