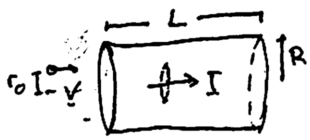
J11E.3

a.) $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$

$$B_\phi \cdot 2\pi r = \mu_0 \frac{I}{\pi R^2} \pi r^2$$



$$\vec{B} = \frac{\mu_0 I r}{2\pi R^2} \hat{\phi}$$

b.) Assuming short cylinder, we can calculate the perpendicular momentum change of the particle as:

$$\vec{F} = -\frac{\mu_0 I r}{2\pi R^2} \cdot q \cdot v \, \hat{r}$$

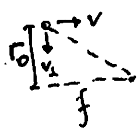
then

$$\Delta P_r \approx -\frac{\mu_0 I r_0}{2\pi R^2} q \cdot v \cdot \Delta t$$

$$= -\frac{\mu_0 I r_0}{2\pi R^2} q \cancel{v} \cdot \frac{L}{\cancel{v}}$$

$$= -\frac{\mu_0 I q r_0 L}{2\pi R^2}$$

the focal length is then given by



$$f = v \cdot \frac{r_0}{v_\perp}$$

$$\approx v \cdot \frac{2\pi R^2 m}{\mu_0 I q L}$$

Here I assume $v_\perp$ is non-relativistic

c.) For 10 GeV/c proton, it's ultrarelativistic $\Rightarrow v = 0.996\,c$ then we want $\tan^{-1} \frac{R}{f} \approx 50\,\text{mrad}$

thus

$$\frac{R}{f} \approx 5 \times 10^{-2}$$

$$\frac{\mu_0 I q L}{2\pi R m v} \approx 5 \times 10^{-2}$$

$$I = 5.17 \times 10^4 \text{ A}$$