Prelims Solutions

Problem J10T2

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1

Consider a gas flowing in the x direction with $v_x = v_x(z)$. Viscosity η is defined by $P_{zx} = -\eta\frac{\partial v_x}{\partial z}$ where P_{zx} is the change of the x-momentum in the z direction per unit time per unit area, which is a stress. Consider patch of area A in a $z = constant$ plane and let $n = \overline{n}, v = \overline{v}$. The number of particles that flow through that patch in time Δt from below (or above) is $\frac{1}{6}nvA\Delta t$ since 1/3 of particles will be going in the z direction and 1/2 of those will be moving in the plus (minus) z direction. A particle with mass m that came from below last had a collision roughly a mean free path l below the plane (i.e. at $z - l$) and thus has x momentum $mv_x(z - l)$. A particle coming from above roughly has x momentum $mv_x(z + l)$. Thus,

$$P_{zx} = \frac{\frac{1}{6}nvA\Delta tmv_x(z-l) - \frac{1}{6}nvA\Delta tmv_x(z+l)}{A\Delta t} = -\frac{1}{3}nvlm\frac{\partial v_x}{\partial z}$$

Which gives $\eta = \frac{1}{3}nvlm$ to first order in the taylor expansion of v_x. Here I think the numerical factor in front shouldn't be trusted, but scaling should be correct.

2

For hard spheres, the cross section doesn't change with temperature (it does in a plasma for ex.). Thus $l = \frac{1}{n\sigma} \to \eta \approx \frac{mv}{\sigma} \approx \frac{(mk_BT)^{\frac{1}{2}}}{\sigma}$. Thus the viscosity does not depend on pressure at constant T! Interactions become non-negligible at higher pressures.

3

Thermal wavelength is $E = \frac{p^2}{2m} = \frac{h^2}{\lambda^2 m} \approx k_BT \to \frac{1}{\lambda} \approx \frac{\sqrt{mk_BT}}{h} \approx \frac{mv}{h}$ where $mv^2 \approx k_BT$ from equipartition. So $\eta \approx \frac{hnl}{\lambda}$.

Using $Z = (\frac{V}{\lambda^3})^N/N!$ I get $s = S/V = n(ln(\frac{1}{n\lambda^3}) + \frac{5}{2})$

4

$\frac{\eta}{s} = \frac{hl}{\lambda(ln(\frac{1}{n\lambda^3})+\frac{5}{2})} \approx h$ when all the scales become similar.