Prelims Solutions

Problem J10M1

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The coordinates of the COM are $x = scos\alpha + \frac{l}{2}sin\theta$, $y = -(ssin\alpha + \frac{l}{2}cos\theta)$ (my θ is negative of theirs). The Lagrangian of the system is $L = T - U = KE_{COM} + KE_{Rotation} - mgy \rightarrow$

$$L = \frac{1}{2}m(\dot{s}^2 + (\frac{l}{2})^2\dot{\theta}^2 + \dot{s}l\dot{\theta}cos\theta cos\alpha - \dot{s}l\dot{\theta}sin\theta sin\alpha) + \frac{1}{2}I\dot{\theta}^2 + mg(ssin\alpha + \frac{l}{2}cos\theta)$$

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Fixed θ means $\dot{\theta} = 0$ and $\ddot{\theta} = 0$ in the EL equations. The EL equation for s is :

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{s}} = \frac{d}{dt}(m\dot{s} + l\dot{\theta}cos\theta cos\alpha - l\dot{\theta}sin\theta sin\alpha) = m\ddot{s} + \ddot{\theta}(...) + \dot{\theta}(...) = \frac{\partial L}{\partial s} = mgsin\alpha \rightarrow \ddot{s} = gsin\alpha$$

The EL equation for θ is :

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} = (m(\frac{l}{2})^2 + I)\ddot{\theta} + m\ddot{s}(lcos\theta cos\alpha - l\sin\theta sin\alpha) + m\dot{s}\dot{\theta}(...) = \frac{\partial L}{\partial \theta} = \dot{s}\dot{\theta}(...) - mg\frac{l}{2}sin\theta \rightarrow \ddot{s}(cos\theta cos\alpha - \sin\theta sin\alpha) = -g\frac{1}{2}sin\theta$$

A consistent solution to these equations requires $sin\alpha(cos\theta cos\alpha - \sin\theta sin\alpha) = -\frac{1}{2}sin\theta$ which gives $tan\theta = \frac{2sin\alpha cos\alpha}{2sin\alpha^2 - 1} = -tan(2\alpha) \rightarrow \theta = -2\alpha$.

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