Trapping atom in an optical lattice

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PROBLEM

A 1-D optical lattice uses counter-propagating light beams to produce a standing wave, which has a periodic intensity pattern. Through the AC Stark-shift effect, the system energy is a function of the positions of the atoms. To simplify the problem, let's consider a hypothetical two-level atom with a ground state $|1\rangle$ and an excited state $|2\rangle$. The energy difference between the two states is ω_0. Now an optical field $E(x,t) = E_0 \cos kx \cos \omega t$ is introduced, and most of the atoms are in state $|1\rangle$ right before the optical field is turned on. The frequency ω is close to ω_0. The optical field interacts with atoms through the electric dipole coupling $\langle 2|E \cdot D|1\rangle = E \cdot \langle D\rangle$. Here, D is the dipole operator, and $|E \cdot \langle D\rangle| \ll |\omega_0 - \omega|$.

a) Please find the perturbed energy of state $|1\rangle$ due to the optical field as a function of position. How deep is the potential well produced by the optical lattice?

b) Where do the atoms tend to be trapped at a different optical frequency ω?

SOLUTION TO PART A

From the setting of the problem we can easily find the Hamiltonian is the following matrix:

$$H = \begin{pmatrix} \hbar\omega_0 & E_0 \cdot \langle D\rangle \cos kx \cos \omega t \\ E_0 \cdot \langle D\rangle \cos kx \cos \omega t & 0 \end{pmatrix} = \hbar\omega_0 \mathcal{P}_2 + \frac{1}{4} E_0 \cdot \langle D\rangle \cos kx \left(\sigma^+ + \sigma^-\right)\left(e^{i\omega t} + e^{-i\omega t}\right) \tag{1}$$

in which $\mathcal{P}_2 = |2\rangle\langle 2|$ is the projection operator. We can find that this Hamiltonian is time-dependent, so it is not possible to find the eigenenergy of the perturbed Hamiltonian. We can do some transformation to get a time-independent Hamiltonian and then solve the energy. Define a ttime-dependent unitary transformation $U = e^{-i\omega \mathcal{P}_2} = \mathcal{P}_1 + e^{-i\omega t}\mathcal{P}_2$, and we do the following transformation which can keep the Schrödinger equation unchanged:

$$H \to U^\dagger H U - i\hbar U^\dagger \partial_t U$$
$$|\psi\rangle \to U^\dagger |\psi\rangle$$

in this problem it is easy to check that $-i\hbar U^\dagger \partial_t U = -\hbar\omega \mathcal{P}_2$, and then the transformed Hamiltonian is

$$\begin{aligned} \tilde{H} &= \hbar(\omega_0 - \omega)\mathcal{P}_2 + \frac{1}{4} E_0 \cdot \langle D\rangle \cos kx \left(\sigma^+ e^{-i\omega t} + \sigma^- e^{i\omega t}\right)\left(e^{i\omega t} + e^{-i\omega t}\right) \\ &= \hbar(\omega_0 - \omega)\mathcal{P}_2 + \frac{1}{4} E_0 \cdot \langle D\rangle \cos kx \left(\sigma^+ + \sigma^- + \sigma^+ e^{-2i\omega t} + \sigma^- e^{2i\omega t}\right) \end{aligned} \tag{2}$$

Now we use the so called "rotating wave approximation": neglect the terms with frequency 2ω because they correspond to the imaginary process which excite the electron to $|2\rangle$ and emit a photon at the same time. This has a smaller contribution

from the second order perturbation theory. Thus the Hamiltonian is time-imdependent after the approximation and it looks like

$$\tilde{H} = \hbar(\omega_0 - \omega)\mathcal{P}_2 + \frac{1}{4}E_0 \cdot \langle D \rangle \cos kx \left(\sigma^+ + \sigma^-\right) \tag{3}$$

Since we know that $E_0 \cdot \langle D \rangle \ll \hbar|\omega_0 - \omega|$, that means we can use non-degenerate 2nd order perturbation theory:

$$E_g^{(2)} = \frac{|E_0 \cdot \langle D \rangle|^2 \cos^2 kx}{4\hbar(\omega - \omega_0)} .$$

SOLUTION TO PART B

We can treat this perturbation energy as a potential. So obviously when $\omega > \omega_0$, points $x_m = (2n+1)\pi/2k\,(n \in \mathbb{Z})$ are the mininum positions of the potential; when $\omega < \omega_0$, $x_c = n\pi/k\,(n \in \mathbb{Z})$ are the minimun points. Atoms will be trapped around these points.