j09e2

a. $\frac{\partial \rho}{\partial t} + \nabla \cdot J = 0.$ $\quad \frac{\partial \rho}{\partial t} = 0$ (steady state) $\rightarrow \nabla \cdot J = 0 \rightarrow J_0$ constant.

$$J_0 = \rho(x)\, v(x)$$

Poisson: $\nabla^2 \phi(x) = \frac{-\rho(x)}{\varepsilon_0}$

also conserve energy: $\frac{1}{2} m v(x)^2 - e\phi(x) = E = 0$ at $x = 0$.

$$\rightarrow \quad v(x) = \sqrt{\frac{2e}{m}}\, \phi^{1/2}(x)$$

$$\nabla^2 \phi(x) = -\frac{J_0}{\varepsilon_0 v(x)} \rightarrow \phi'' = -\underbrace{\frac{J_0}{\varepsilon_0}\sqrt{\frac{m}{2e}}}_{\alpha}\, \phi^{-1/2}$$

try $\phi = A x^n$

$$A n(n-1) x^{n-2} = -\frac{\alpha}{\sqrt{A}} x^{-n/2} \rightarrow n - 2 = -\frac{n}{2} \rightarrow n = \frac{4}{3}$$

and $A^{3/2}\left(\frac{4}{3}\right)\left(\frac{1}{3}\right) = -\alpha \rightarrow A = \left(\frac{9}{4}\alpha\right)^{2/3}$

so $\phi(x) = \left(\frac{9}{4}\alpha\right)^{2/3} x^{4/3}$ with $\alpha = \frac{J_0}{\varepsilon_0}\sqrt{\frac{m}{2e}}$

so $\phi(a) = \left(\frac{9}{4}\alpha\right)^{2/3} a^{4/3} = V_0$

b. $$\left(\frac{9}{4\varepsilon_0}\sqrt{\frac{m}{2e}}\right)^{2/3} J^{2/3} a^{4/3} = V_0 \rightarrow J = V_0^{3/2}\left(\frac{4\varepsilon_0}{9a^2}\sqrt{\frac{2e}{m}}\right)$$

c. notice $J \sim \frac{1}{\sqrt{m}}$ so $\frac{J_{Au}}{J_p} = \sqrt{\frac{m_p}{M_{Au}}} \approx \sqrt{\frac{1}{A}} = \frac{1}{\sqrt{79}} \approx \frac{1}{8.9}$