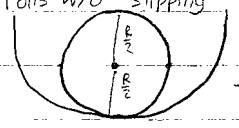
Jan 2008 #2 (CM)

rolls w/o slipping



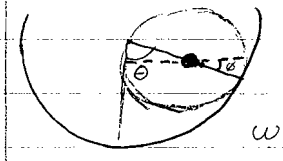
$L = T - U$

$T = \frac{1}{2}mV^2 + \frac{1}{2}I\omega^2 \qquad U = mgy$

V = speed of center of mass

I = moment of inertia for a sphere, $I = \frac{2}{5}m\left(\frac{R}{2}\right)^2 = \frac{1}{10}mR^2$

ω = rotational speed about center of mass



$\phi = \frac{\pi}{2} - \theta \qquad \dot{\phi} = -\dot{\theta}$

$\omega = \dot{\phi} = -\dot{\theta}$

rolling without slipping condition: $V = \frac{R}{2}\omega = -\frac{R}{2}\dot{\theta}$

$T = \frac{1}{2}m\left(\frac{R}{2}\dot{\theta}\right)^2 + \frac{1}{2}I\dot{\theta}^2 = \frac{1}{8}mR^2\dot{\theta}^2 + \frac{1}{20}mR^2\dot{\theta}^2 = \frac{7}{40}mR^2\dot{\theta}^2$

$y = -\frac{R}{2}\cos\theta$ coordinate of center of mass

$U = -\frac{mgR}{2}\cos\theta$

$L = \frac{7}{40}mR^2\dot{\theta}^2 + \frac{mgR}{2}\cos\theta$

$\frac{\partial L}{\partial \dot{\theta}} = \frac{7}{20}mR^2\dot{\theta} \qquad \frac{\partial L}{\partial \theta} = -\frac{mgR}{2}\sin\theta$

$\frac{7}{20}mR^2\ddot{\theta} = -\frac{mgR}{2}\sin\theta \qquad \frac{7}{10}R\ddot{\theta} = -g\sin\theta$

$\ddot{\theta} = -\frac{10g}{7R}\sin\theta$

small θ: $\sin\theta \approx \theta \qquad \ddot{\theta} = -\frac{10g}{7R}\theta$

$\omega_{osc} = \left(\frac{10g}{7R}\right)^{1/2}$