

Prelim solutions - January 2008

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1 Mechanics

1.1 Problem 2

Let θ be the angle between the vertical and the line that joins the center of the cylinder and the center of the sphere. Let ϕ be the angle by which the sphere rotates about its axis as it moves inside the cylinder. Because the sphere rolls without slipping, we must have:

$$R\theta = \frac{R}{2}\phi$$

We can now write the Lagrangian:

$$\mathcal{L} = \frac{1}{2}m \left(\frac{R}{2}\right)^2 \dot{\theta}^2 + \frac{1}{2}I(2\dot{\theta})^2 - mg \left(R - \frac{R}{2}\cos\theta\right) = \frac{13}{40}mR^2\dot{\theta}^2 - mgR \left(1 - \frac{1}{2}\cos\theta\right)$$

$$\frac{\partial \mathcal{L}}{\partial \dot{\theta}} = \frac{13}{20}mR^2\dot{\theta}$$

$$\frac{\partial \mathcal{L}}{\partial \theta} = -mgR\frac{1}{2}\sin\theta$$

$$\frac{13}{10}R\ddot{\theta} \approx -g\theta$$

$$\omega = \sqrt{\frac{10g}{13R}} \tag{1}$$