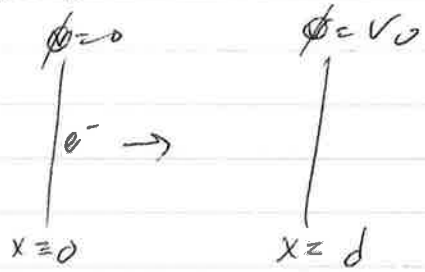
some mistakes

jobe 3



$$\nabla^2 \phi = \frac{-\rho(x)}{\varepsilon_0}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot J = 0 \rightarrow 0 \quad \rightarrow \quad J_0 = \rho(x) v(x) \quad \text{independent of } x.$$

a - single electron, conserve energy: $\frac{1}{2} m v^2 - e\phi(x) = 0$

$$\phi(x) = \frac{e v^2}{2m} = \frac{e}{2m} \frac{J_0^2}{\rho(x)^2}$$

$$\rightarrow \quad \rho(x) = J_0 \sqrt{\frac{e}{2m}} \phi^{-1/2}(x)$$

Poisson: $\phi''(x) = -\frac{\rho(x)}{\varepsilon_0} = -\frac{J_0}{\varepsilon_0}\sqrt{\frac{e}{2m}}\,\phi^{-1/2}(x)$

$$\phi''(x) = -a\phi^{-1/2}(x) \qquad a = \frac{J_0}{\varepsilon_0}\sqrt{\frac{e}{2m}}$$

guess: $\phi(x) = Ax^n$

$$\phi''(x) = nA n(n-1) x^{n-2} = -a x^{-1/2} \rightarrow n = 3/2$$

$$A^{3/2}\left(\frac{3}{2}\right)\left(\frac{1}{2}\right) = -a \rightarrow A = \left(-\frac{4a}{3}\right)^{2/3}$$

$$\phi(x) = \left(\frac{4a}{3}\right)^{2/3} x^{3/2} \quad \text{with} \quad a = \frac{J_0}{\varepsilon_0}\sqrt{\frac{e}{2m}}$$

b. $V_0 = \phi(d) = \left(\frac{4a}{3}\right)^{2/3} d^{3/2} = \left(\frac{4}{3}\right)^{2/3}\left(\frac{e}{2m\varepsilon_0^2}\right)^{1/3} J_0^{2/3} d^{3/2}$

$\rightarrow \quad J_0 = V_0^{3/2}\left(\frac{3}{4}\right)\left(\frac{e}{2m\varepsilon_0}\right)^{1/2} d$