
January 2008 Preliminary Exam, Electromagnetism Problem 2

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Problem:

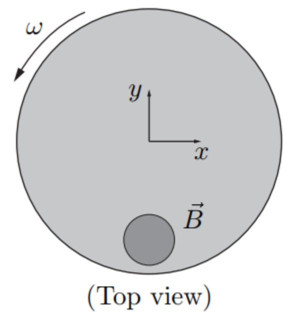

Figure 1: *Figure from J08E.2*

An aluminum disk of radius R, thickness d, conductivity σ, and mass density ρ is mounted on a frictionless vertical axis. It passes between the poles of a magnet near its rim which produces a B-field perpendicular to the plane of the disk over a small area A of the disk. The initial speed of the disk is $\omega(t=0)=\omega_0$.

(a) An observer on the disk, moving between the pole pieces of the magnet would feel an electric field. Give the direction and magnitude of this field in terms of R, ω_0, and B (assume the angular speed ω is small enough so that $\gamma \sim 1$). This results in a current density.

(b) Calculate the torque due to the Lorentz force produced on this current density by the $\vec{B}$-field of the magnet.

(c) Given the moment of inertia of the disk around its axis ($I=\frac{1}{2}MR^2$), write out the equation of motion of the disk and calculate the number of revolutions of the disk before it comes to rest.

Reason for Solution:

In Andy's solution, he performs the u-substitution incorrectly. This answer will also give a bit more detail.

Solution:

(a) The fastest way for me to do this is to calculate the force an electron would feel in the lab frame due to the magnetic field and recall that this same force will be felt in the observer's frame due to an electric field. The Lorentz force felt by an electron, from the lab frame, is the following:

$$\vec{F}_{lab} = q\vec{v}\times\vec{B} = -e(\omega R)B\hat{y} \tag{1}$$

Set this equal to whatever the electric field is in the observer's frame:

$$\vec{F}_{obs} = q\vec{E} = -e\vec{E} = -e\omega RB\hat{y} \implies \vec{E} = \omega RB\hat{y} \tag{2}$$

(b) This electric field $\vec{E}=\omega RB\hat{y}$ induces a current density $\vec{J}=\sigma\vec{E}$. This will cause a force on the disc where current is moving according to $\vec{F}=\vec{I}L\times\vec{B}=\vec{J}Ad\times\vec{B}$. This force points circumferentially on the disc, causing a torque $\vec{\tau}=\vec{R}\times\vec{F}$:

$$\vec{\tau} = -RAd\sigma\omega RB\hat{z} \tag{3}$$

(c) Given the moment of inertia $I = \frac{1}{2}MR^2$, I can calculate the angular acceleration on the disc due to this torque:

$$\tau = I\dot{\omega} = \frac{1}{2}(\pi R^2 d\rho)\dot{\omega} = -RAd\sigma\omega RB \tag{4}$$

$$\implies \dot{\omega} = \frac{-2A\sigma B}{\pi\rho R^2}\omega \tag{5}$$

$$\implies \omega(t) = \omega_0 \exp\left(\frac{-2A\sigma B}{\pi\rho R^2}t\right) \tag{6}$$

To calculate the number of rotations, integrate this with respect to t and divide by the 2π radians per rotation:

$$N = \frac{1}{2\pi}\int_0^\infty \omega_0 \exp\left(\frac{-2A\sigma B}{\pi\rho R^2}t\right) dt = \frac{\omega_0}{2\pi}\frac{\pi\rho R^2}{2A\sigma B}\int_0^\infty e^{-u}\, du = \frac{\omega_0\rho R^2}{4A\sigma B} \tag{7}$$