J07T1 / useful for M02T2

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January 2018

1 Introduction

Pablo Mostiero has an excellent solution for J07T1. However, I wasn't sure why the equation

$$-\epsilon + \mu_H(\epsilon = 0) = \mu_P + \mu_e \tag{1}$$

was true. I figured it might be helpful to have a brief explanation of this equation.

2 Chemical Potential of Plasmas

The macrostate of an isolated state in equilibrium will be the one which maximizes entropy (duh!). When particles can be exchanged between systems A and B, where losing a particle from A adds a particle to B, we have that

$$\frac{\partial S_{total}}{\partial N_A} = 0 = \left(\frac{\partial S_A}{\partial N_A}\right)_{E,V} - \left(\frac{\partial S_B}{\partial N_B}\right)_{E,V} \tag{2}$$

where the derivatives are with respect to fixed E and V. I skipped a few steps because this derivation is done in textbooks commonly. This tells us that the chemical potentials, $\mu_i = -T(\frac{\partial S_i}{\partial N_i})_{E,V}$ are equal for the systems.

This changes slightly in a hydrogen plasma because when we lose a neutral atom, we create two new particles (an electron and proton). Thus, we can redo our derivation to get

$$\frac{\partial S_{total}}{\partial N_H} = 0 = \left(\frac{\partial S_H}{\partial N_H}\right)_{E,V} + \left(\frac{\partial S_P}{\partial N_H}\right)_{E,V} + \left(\frac{\partial S_e}{\partial N_H}\right)_{E,V} \tag{3}$$

$$= \left(\frac{\partial S_H}{\partial N_H}\right)_{E,V} - \left(\frac{\partial S_P}{\partial N_P}\right)_{E,V} - \left(\frac{\partial S_e}{\partial N_e}\right)_{E,V} \tag{4}$$

So we have $\mu_H = \mu_P + \mu_e$. But the chemical potential of hydrogen is related to the change in entropy with changes in N_H at *fixed* E. When we add a hydrogen atom, because it has bound energy $-\epsilon$, we need to account for that change in energy. (When we add a proton or an electron, because the binding energy of these is 0, we can just use the chemical potential of an ideal gas) The chemical potential is also defined as $\left(\frac{\partial E}{\partial N}\right)_{V,S}$. Since we increase (actually, decrease) the energy by $-\epsilon$ when a proton and an electron recombine, then the chemical potential of hydrogen is whatever the ideal gas chemical potential of hydrogen is, plus $-\epsilon$. Thus we have our equation

$$-\epsilon + \mu_H(\epsilon = 0) = \mu_P + \mu_e \tag{5}$$

The rest of the solution for J07T1 is straightforward, which Pablo does a nice job of.