J07T1 / useful for M02T2

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1 Introduction

Pablo Mostiero has an excellent solution for J07T1. However, I wasn’t sure why the equation

\[-\epsilon + \mu_H(\epsilon = 0) = \mu_P + \mu_e\]  

was true. I figured it might be helpful to have a brief explanation of this equation.

2 Chemical Potential of Plasmas

The macrostate of an isolated state in equilibrium will be the one which maximizes entropy (duh!). When particles can be exchanged between systems A and B, where losing a particle from A adds a particle to B, we have that

\[
\frac{\partial S_{\text{total}}}{\partial N_A} = 0 = \left( \frac{\partial S_A}{\partial N_A} \right)_{E,V} - \left( \frac{\partial S_B}{\partial N_B} \right)_{E,V}
\]

where the derivatives are with respect to fixed $E$ and $V$. I skipped a few steps because this derivation is done in textbooks commonly. This tells us that the chemical potentials, $\mu_i = -T\left( \frac{\partial S_i}{\partial N_i} \right)_{E,V}$ are equal for the systems.

This changes slightly in a hydrogen plasma because when we lose a neutral atom, we create two new particles (an electron and proton). Thus, we can redo our derivation to get

\[
\frac{\partial S_{\text{total}}}{\partial N_H} = 0 = \left( \frac{\partial S_H}{\partial N_H} \right)_{E,V} + \left( \frac{\partial S_P}{\partial N_P} \right)_{E,V} + \left( \frac{\partial S_e}{\partial N_e} \right)_{E,V}
\]

So we have $\mu_H = \mu_P + \mu_e$. But the chemical potential of hydrogen is related to the change in entropy with changes in $N_H$ at fixed $E$. When we add a hydrogen atom, because it has bound energy $-\epsilon$, we need to account for that change in energy. (When we add a proton or an electron, because the binding energy of
these is 0, we can just use the chemical potential of an ideal gas. The chemical potential is also defined as \( \frac{\partial E}{\partial N} \). Since we increase (actually, decrease) the energy by \(-\epsilon\) when a proton and an electron recombine, then the chemical potential of hydrogen is whatever the ideal gas chemical potential of hydrogen is, plus \(-\epsilon\). Thus we have our equation

\[
-\epsilon + \mu_H(\epsilon = 0) = \mu_P + \mu_e
\]  

(5)

The rest of the solution for J07T1 is straightforward, which Pablo does a nice job of.