1 Quantum

1.1 Problem 2

(a) A discontinuity in the wavefunction would mean there is a step function in the wavefunction, which would mean a delta function in the kinetic energy (derivative of the wavefunction). This is an infinite kinetic energy, which is unacceptable. Thus, we write the basic condition:

$$\psi_{left} = \psi_{right} \tag{1}$$

Deriving the other condition should be very simple. The Schrödinger equation reads:

$$\int_{-\epsilon}^{\epsilon} \frac{1}{2} p(m(x))^{-1} p\psi \, dx + \int_{-\epsilon}^{\epsilon} V(x)\psi \, dx = \int_{-\epsilon}^{\epsilon} E\psi \, dx$$

Since all V,E,ψ are finite, the only remaining term is the first one:

$$m(0^-)^{-1}\frac{\partial \psi}{\partial x}(0^-) = m(0^+)^{-1}\frac{\partial \psi}{\partial x}(0^+) \tag{2}$$

(b) The easiest quantity to get is the k quantity. Just apply the Hamiltonian on both sides:

$$\frac{1}{2} p(m_0)^{-1}(-i\hbar)\frac{\partial \psi}{\partial x} = E\psi$$

$$\frac{1}{2} p(m_0)^{-1}(-i\hbar)kAcos[k(x - x_0)] = E\psi$$

$$\frac{1}{2}(\hbar^2/m_0)k^2 Asin[k(x - x_0] = EAsin[k(x - x_0)]$$

$$k = \frac{\sqrt{2m_0 E}}{\hbar} \tag{3}$$

Solve the Schrödinger equation on the left side of the discontinuity:

$$-\frac{\hbar^2}{2m_1}$$