1 Mechanics

1.1 Problem 2

The kinetic energy term due to m is a little tricky. To get it correctly, we should write the coordinates of m (X and Y) in terms of x and θ. A little bit of trigonometry and looking at the picture tells you that:

$$X = x cos\theta - L sin\theta$$
$$Y = L cos\theta - x sin\theta$$

$$\dot{X} = \dot{x} cos\theta - x sin\theta\dot{\theta} - L cos\theta\dot{\theta}$$
$$\dot{Y} = -L sin\theta\dot{\theta} + \dot{x} sin\theta + x cos\theta\dot{\theta}$$

$$\dot{X}^2 + \dot{Y}^2 = \dot{x}^2 + x^2\dot{\theta}^2 + L^2\dot{\theta}^2 - 2\dot{x}L\dot{\theta}$$

Since we want the linearized equations of motion, we want the lagrangian to contain terms of order up to 2. Thus,

$$\dot{X}^2 + \dot{Y}^2 \approx \dot{x}^2 + L^2\dot{\theta}^2 - 2\dot{x}L\dot{\theta}$$

$$\mathcal{L} = \frac{1}{2}ML^2\dot{\theta}^2 + \frac{1}{2}m(\dot{x}^2 + L^2\dot{\theta}^2 - 2\dot{x}L\dot{\theta}) + (M+m)gL cos\theta + mgx sin\theta$$

Applying the Euler-Lagrange equations gives the equations of motion:

$$(M+m)L^2\ddot{\theta} - m\ddot{x}L = -(M+m)gL\theta + mgx \tag{1}$$
$$-L\ddot{\theta} + \ddot{x} = g\theta \tag{2}$$

These two equations can be written together as a matrix equation:

$$\boldsymbol{M}\ddot{\boldsymbol{\xi}} = -\boldsymbol{K}\boldsymbol{\xi}$$

where ξ has components x and θ. We will guess that the solution is oscillatory, and so to get the eigenfrequencies, we have to solve the trascendental equation:

$$det(\boldsymbol{K} - \omega^2\boldsymbol{M}) = 0$$

I get two solutions from this equation $\omega_1^2 = g/L$ and $\omega_2^2 = -2mg/(ML)$. I was really careful in writing the matrices and the characteristic equation, so I don't know how I reached this problem. It could be, also, that my solution is correct, the negative ω^2 meaning that the corresponding eigenmode is of exponential form, but I don't think so. I'll wait for opinions.