# 1 J06T3

## 1.1 (a)

$$Z = \sum_{0}^{L/d} e^{-n\epsilon/kT} \approx \frac{1}{1 - e^{-\beta\epsilon}}$$

where I have used the assumption that since we expect $\bar{n}$ to be of order of magnitude 1 or greater, and since $\bar{n}d << L$, then $d << L$, so that we can take the upper limit of the sum to $\infty$.

$$\bar{n} \approx \sum_{0}^{\infty} n \frac{e^{-n\epsilon/kT}}{Z} = -\frac{1}{\epsilon}\frac{\partial(lnZ)}{\partial\beta} = \frac{1}{e^{\beta\epsilon} - 1}$$

$$\bar{n} \approx \frac{kT}{\epsilon} \tag{1}$$

where I used the given assumption $kT >> \epsilon$. As we can see, $\bar{n} > 1$, so that our first assumption was justified.

After the force is applied, the energy becomes:

$$H = n\epsilon - 2Fnd \tag{2}$$

Here's where I start doubting. I'm not sure that this minus sign is correct. Can one of you guys explain it to me please?

A similar approach to what we did before gives:

$$\bar{n} = \frac{1}{e^{\beta(\epsilon - 2Fd)} - 1} \tag{3}$$

When $2Fd = \epsilon$, this becomes infinite, which makes sense because at that point the applied force can release each of the links. When $F < \frac{\epsilon}{2d}$, the occupation number gets smaller as F gets smaller, and this also makes sense. So I come to the conclusion that (probably) the minus sign above was correct.

## 1.2 (b)

$$Z \approx \sum_{0}^{\infty} e^{-n\beta(\epsilon - 2Fd)} d(n)$$

where $d(n)$ is the degeneracy of the state. When a link is removed, each of its two hanging parts can go into any of $g$ states, so the degeneracy of the removed link is $g^2$. For n links broken, there are 2n distinguishable "things" each of which can go into $g$ different states. Thus, $d(n) = g^{2n}$

$$Z \approx \sum_{0}^{\infty} e^{-n\beta(\epsilon - 2Fd)} g^{2n} = \frac{1}{1 - g^2 e^{-\beta(\epsilon - 2Fd)}} \tag{4}$$

where I have made an <u>ASSUMPTION</u> that $g^2 < e^{\beta(\epsilon - 2Fd)}$, for otherwise the series diverges.

$$F = -kTlnZ = kTln[1 - g^2 e^{-\beta(\epsilon - 2Fd)}] \tag{5}$$

Since we are assuming $N = L/d = \infty$, when all of the links are open, there will be an infinite number of "accessible states", i.e., the partition function will be infinite. Hence,

$$1 - g^2 e^{-\beta_C(\epsilon - 2Fd)} = 0$$
$$T_C = \frac{\epsilon - 2Fd}{2klng} \tag{6}$$

## 1.3 (c)

Going back to the expression for the partition function (4), and writing the formula for the average value of n as before, we get:

$$\bar{n} = -\frac{1}{\epsilon - 2Fd}\frac{\partial(lnZ)}{\partial\beta} = \frac{1}{g^{-2}e^{\beta(\epsilon-2Fd)} - 1} \tag{7}$$

To graph this function, notice that it is 0 at T=0, and infinite at $T = T_C$. Also look at the derivative:

$$\frac{\partial\bar{n}}{\partial T} = \frac{1}{[g^{-2}e^{\beta(\epsilon-2Fd)} - 1}g^{-2}e^{\beta(\epsilon-2Fd)}\frac{\beta(\epsilon - 2Fd)}{T}$$

Notice that this is always positive, so we can conclude that the function is monotonically increasing. We thus make a graph that is increasing with some sort of curved shape from 0 at T=0 to $\infty$ at $T = T_C$.