1 Electromagnetism

1.1 Problem 3

(a) The circular trajectory can be interpreted as three dipoles oscillating in phase. Since the dipoles are perpendicular to each other, we don't need to get the fields for each and compute the Poynting vector from that. Rather, we can just sum the powers of the three dipoles. For any one of them, the power radiated through a spherical surface at a radius r from the origin is:

$$P = \frac{1}{4\pi\epsilon_0}\frac{q^2R^2\omega^4}{3c^3}$$

This formula is derived in Griffiths for a general dipole, or it can also be obtained from the Larmor formula, after averaging over time. R is the radius of our circle. Since we have three dipoles, our real power is:

$$P_{circle} = \frac{1}{4\pi\epsilon_0}\frac{q^2R^2\omega^4}{c^3}$$

Now notice that at any extremum point, the kinetic energy of the system is 0, while the potential energy is:

$$U = \frac{3}{2}KR^2 = E$$

by conservation of energy Solve for R^2:

$$R^2 = \frac{2E}{3K}$$

And also use:

$$\omega^2 = \frac{K}{m}$$

Plugging everything in, with the fact that the power is the energy radiated out per unit time:

$$\frac{dE}{dt} = -\frac{1}{4\pi\epsilon_0}\frac{q^2 2EK^2}{3Km^2c^3}$$

Thus the characteristic time is:

$$\tau = \frac{6\pi\epsilon_0 m^2c^3}{q^2K} \tag{1}$$

(b) We want the characteristic decay time to be much larger than the time it takes for the particle to go around the circle once, assuming that the particle travels at an angular velocity of ω:

$$\frac{2\pi}{\omega} << \frac{6\pi\epsilon_0 m^2c^3}{q^2K}$$

$$\frac{3\epsilon_0 m^{3/2}c^3}{q^2K^{1/2}} >> 1 \tag{2}$$

(c) You need to know the formula for the radiation reaction or how to "derive" it (see Griffiths section 9.3). The formula is:

$$\boldsymbol{F_{rad}} = \frac{1}{4\pi\epsilon_0}\frac{2}{3}\frac{q^2}{c^3}\dot{\boldsymbol{a}}$$

In our case, we can assume that the magnitude of the radiation reaction force is about 3 times the magnitude of the force due to each of the dipoles:

$$F_{rad} \approx \frac{1}{4\pi\epsilon_0}2\frac{q^2}{c^3}R\omega^3$$

which by the previous part satisfies:

$$F_{rad} << RK \approx F_{spring} \tag{3}$$